Intermediate Microeconomics

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Contents

1 Market

1.1 Constructing a Model

Economics proceeds by developing models of social phenomena. By a model we mean a simplified representation of reality.

❖ **Definition:1.1 Exogenous Variable and Endogenous Variable** ▽

An **exogenous variable** is taken as determined by factors not discussed in this particular model, while an **endogenous variable** is determined by forces described in the model.

1.2 Optimization and Equilibrium

Whenever we try to explain the behavior of human beings we need to have a framework on which our analysis can be based. In much of economics we use a framework built on the following two simple principles.

❏ **Axiom:1.2 The Optimization Principle** ▽

People try to choose the best patterns of consumption that they can afford.

❏ **Axiom:1.3 The Equilibrium Principle** ▽

Prices adjust until the amount that people demand of something is equal to the amount that is supplied.

1.3 Demand Curve, Supply Curve and Market Equilibrium

❖ **Definition:1.4 Reservation Price(保留价格)** ▽

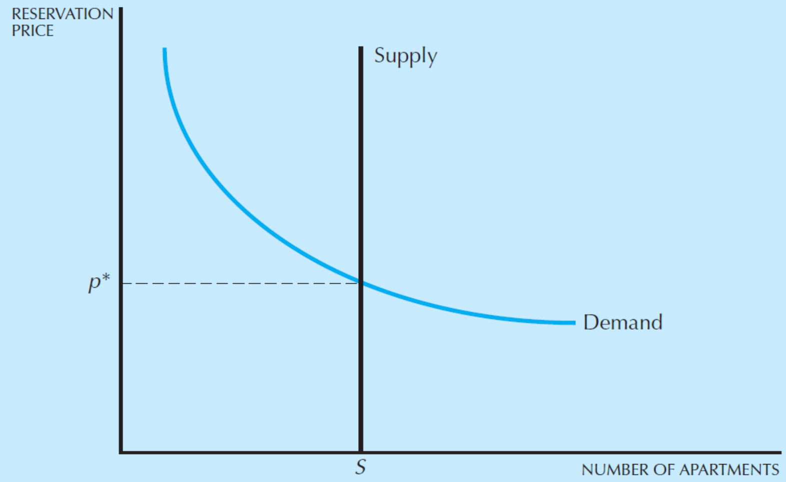
A person's maximum willingness to pay for something that person's **reservation price**.

The reservation price is the highest price that a given person will accept and still purchase the good. In other words, a person's reservation price is the price at which he or she is just indifferent between purchasing or not purchasing the good.

❖ **Definition:1.5 Demand Curve, Supply Curve** ▽

A **demand curve** is a curve that relates the quantity demanded to price.

A **supply curve** is a curve that relates the quantity supplied to price.



Equilibrium in the apartment market. The equilibrium price, p^*, is determined by the intersection of the supply and demand curves.

Figure 1.1: Market Equilibrium

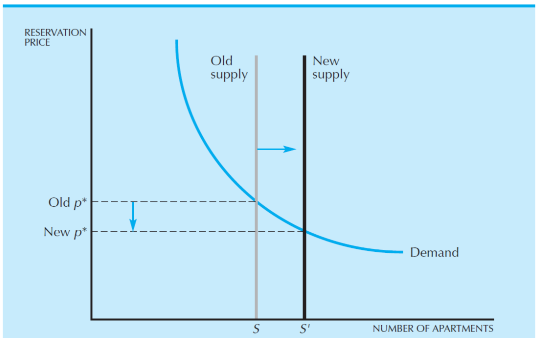
In this graph we have used p^* to denote the price where the quantity of goods demanded equals the quantity supplied. This is the **equilibrium price of goods**. At this price, each consumer who is willing to pay at least p^* is able to find a good to buy, and each seller will be able to sell goods at the going market price. Neither the consumers nor the sellers have any reason to change their behavior. This is why we refer to this as an **equilibrium**: *no change in behavior will be observed.*

1.4 Comparative Statics

Now that we have an economic model of the apartment market, we can begin to use it to analyze the behavior of the equilibrium price. For example, we can ask how the price of apartments changes when various aspects of the market change. This kind of an exercise is known as **comparative statics**, because it involves comparing two “static” equilibria without worrying about how the market moves from one equilibrium to another.

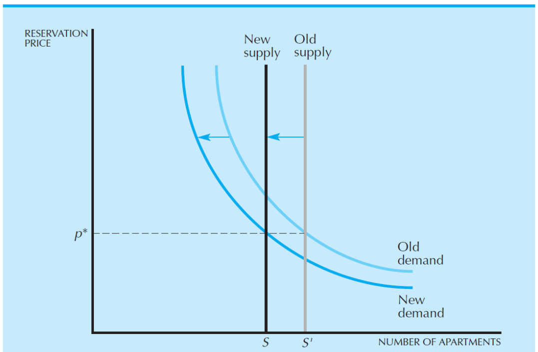
Let's consider another example of a surprising comparative statics analysis: the effect of an apartment tax. Suppose that the city council decides that there should be a tax on apartments of $50 a year. Thus each landlord will have to pay $50 a year to the city for each apartment that he owns. What will this do to the price of apartments?

In fact, the equilibrium price of apartments will remain unchanged! The supply curve doesn't change—there are just as many apartments after the tax as before the tax. And the



Increasing the supply of apartments. As the supply of apartments increases, the equilibrium price decreases.

Figure 1.2: Increase the Supply



Effect of creating condominiums. If demand and supply both shift left by the same amount the equilibrium price is unchanged.

Figure 1.3: Effect of Creating Condominiums

demand curve doesn't change either, since the number of apartments that will be rented at each different price will be the same as well. If neither the demand curve nor the supply curve shifts, the price can't change as a result of the tax.

Here is a way to think about the effect of this tax. Before the tax is imposed, each landlord is charging the highest price that he can get that will keep his apartments occupied. The equilibrium price p^* is the highest price that can be charged that is compatible with all of the apartments being rented. After the tax is imposed, the landlords can not raise their prices to compensate for the tax.If they were charging the maximum price that the market could bear, the landlords couldn't raise their prices any more: none of the tax can get passed along to the renters. The landlords have to pay the entire amount of the tax.

✎ **Remark:1.6 Tax Allocation** ▽

When supply is inelastic, taxation does not affect the equilibrium, and the seller bears all the tax burden.

When demand is inelastic, taxation does not affect the equilibrium, and the consumer bears all the tax burden.

1.5 Other Ways to Allocate Apartments

1.5.1 The Discriminating Monopolist

❖ **Definition:1.7 Monopoly** ▽

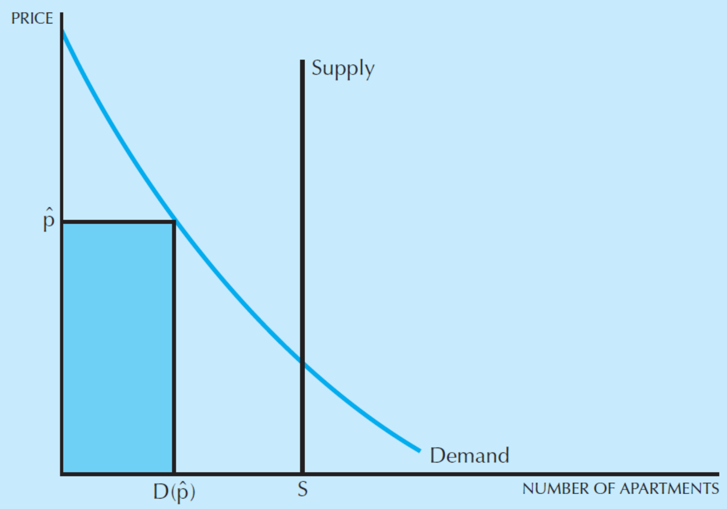
A situation where a market is dominated by a single seller of a product is know as a **monopoly**.

In renting the apartments the landlord could decide to auction them off one by one to the highest bidders. Since this means that different people would end up paying different prices for apartments, we will call this the case of the **discriminating monopolist**.

Here is the interesting feature of the discriminating monopolist: exactly the same people will get the apartments as in the case of the market solution, namely, everyone who valued an apartment at more than p^*. The last person to rent an apartment pays the price p^*—the same as the equilibrium price in a competitive market. The discriminating monopolist's attempt to maximize his own profits leads to the same allocation of apartments as the supply and demand mechanism of the competitive market. *The amount the people pay is different, but who gets the apartments is the same.*

1.5.2 The Ordinary Monopolist

If the seller rents all apartments at the same price, in this case the monopolist faces a tradeoff: if he chooses a low price he will rent more apartments, but he may end up making less than if he sets a higher price. The monopolist will want to restrict the output available



Revenue box. The revenue received by the monopolist is just the price times the quantity, which can be interpreted as the area of the box illustrated.

Figure 1.4: Revenue Box

in order to maximize his profit. This means that the monopolist will generally want to charge a price that is higher than the equilibrium price in a competitive market, p^*. In the case of the ordinary monopolist, fewer apartments will be rented, and each apartment will be rented at a higher price than in the competitive market.

1.5.3 Rent Control

A third and final case that we will discuss will be the case of rent control. Suppose that the government decides to impose a maximum rent that can be charged for apartments, say $p_{\max}$. We suppose that the price $p_{\max}$ is less than the equilibrium price in the competitive market, p^*. If this is so we would have a situation of **excess demand**: there are more people who are willing to rent apartments at $p_{\max}$ than there are apartments available.

Under rent control the same number of apartments will be rented at the rent-controlled price as were rented under the competitive price: they'll just be rented to different people.

1.6 Pareto Efficiency

One useful criterion for comparing the outcomes of different economic institutions is a concept known as **Pareto efficiency** or **economic efficiency**.

❖ **Definition:1.8 Pareto Improvement, Pareto Efficient** ▽

If we can find a way to make some people better off without making anybody else worse off, we have a **Pareto improvement**.

If an allocation allows for a Pareto improvement, it is called **Pareto inefficient**; if an allocation is such that no Pareto improvements are possible, it is called **Pareto efficient**.

A Pareto inefficient allocation has the undesirable feature that there is some way to make somebody better off without hurting anyone else.

Suppose that we think of all voluntary trades as being carried out so that all gains from trade are exhausted. The resulting allocation must be Pareto efficient. If not, there would be some trade that would make two people better off without hurting anyone else—but this would contradict the assumption that all voluntary trades had been carried out. *An allocation in which all voluntary trades have been carried out is a Pareto efficient allocation.*

1.7 Comparing Ways to Allocate Apartments

If there are S apartments to be rented, then the S people with the highest reservation prices end up getting apartments in the inner ring. This allocation is Pareto efficient—anything else is not, since any other assignment of apartments to people would allow for some trade that would make at least two of the people better off without hurting anyone else.

There are no further gains from trade to be had once the apartments have been rented in a competitive market. The outcome of the competitive market is Pareto efficient.

The discriminating monopolist assigns apartments to exactly the same people who receive apartments in the competitive market. Under each system everyone who is willing to pay more than p^* for an apartment gets an apartment. Thus the discriminating monopolist generates a Pareto efficient outcome as well.

✎ **Remark:1.9** ▽

In general, Pareto efficiency doesn't have much to say about distribution of the gains from trade. It is only concerned with the efficiency of the trade: whether all of the possible trades have been made.

The ordinary monopolist who is constrained to charge just one price is not Pareto efficient. Since all the apartments will not in general be rented by the monopolist, he can increase his profits by renting an apartment to someone who doesn't have one at any positive price. There is some price at which both the monopolist and the renter must be better off. As long as the monopolist doesn't change the price that anybody else pays, the other renters are just as well off as they were before. Thus we have found a **Pareto improvement**.

The final case is that of rent control. This also turns out not to be Pareto efficient.

✎ **Remark:1.10** ▽

We have analyzed the equilibrium pricing of apartments in the **short run**—when there is a fixed supply of apartments. But in the **long run** the supply of apartments can change. Just as the demand curve measures the number of apartments that will be demanded at different prices, the supply curve measures the number of apartments that will be supplied at different prices.

2 Budget Constraint

The economic theory of the consumer is very simple: economists assume that consumers choose the best bundle of goods they can afford.

2.1 The Budget Constraint

Suppose that there is some set of goods from which the consumer can choose. In real life there are many goods to consume, but for our purposes it is convenient to consider only the case of two goods. We will indicate the consumer's **consumption bundle** by (x_1, x_2). Sometimes we denote the consumer's bundle by X, where X is an abbreviation for (x_1, x_2).

We suppose that we can observe the prices of the two goods, (p_1, p_2), and the amount of the consumer has to spend, m. Then the budget constraint of the consumer can be written as

$$p_1x_1 + p_2x_2 \leq m. \tag{2.1}$$

We call this set of affordable consumption bundles at prices (p_1, p_2) and income m the **budget set** of the consumer.

✎ **Remark:2.1 Composite Good** ▽

The two-good assumption is general, since we can often interpret one of the goods as representing everything else the consumer might want to consume.
When we adopt this interpretation, it is convenient to think of good 2 as being the dollars that the consumer can use to spend on other goods. Under this interpretation the price of good 2 will automatically be 1. Thus the budget constraint will take the form

$$p_1x_1 + x_2 \leq m. \tag{2.2}$$

We say that good 2 represents a **composite good** that stands for everything else that the consumer might want to consume other than good 1. Such a composite good is measured in dollars to be spent on goods other than good 1.

2.2 Properties of the Budget Set

The **budget line** is the set of bundles that cost exactly m:

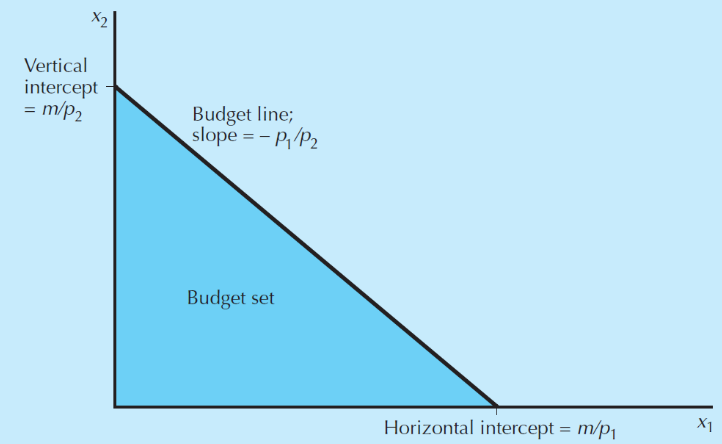
$$p_1x_1 + p_2x_2 = m. \tag{2.3}$$

These are the bundles of goods that just exhaust the consumer's income.

We can rearrange the budget line in the equation above to give us the formula

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1. \tag{2.4}$$

The formula tells us how many units of good 2 the consumer needs to consume in order to just satisfy the budget constraint if he is consuming x_1 units of good 1. The slope of the



The budget set. The budget set consists of all bundles that are affordable at the given prices and income.

Figure 2.1: The Budget Set

budget line measures the rate at which the market is willing to "substitute" good 1 for good 2. Suppose that the consumer is going to increase his consumption of good 1 by $\mathrm{d}x_1$. His consumption of good 2 have to change $\mathrm{d}x_2$ in order to satisfy his Now note that if she satisfies her budget constraint before and after making the change she must satisfy

$$p_1x_1 + p_2x_2 = m.$$

And

$$p_1(x_1 + \mathrm{d}x_1) + p_2(x_2 + \mathrm{d}x_2) = m.$$

Subtracting the first equation from the second gives

$$p_1\mathrm{d}x_1 + p_2\mathrm{d}x_2 = 0.$$

This says that the total value of the change in her consumption must be zero. Solving for $\mathrm{d}x_2/\mathrm{d}x_1$, the rate at which good 2 can be substituted for good 1 while still satisfying the budget constraint, gives

$$\frac{\mathrm{d}x_2}{\mathrm{d}x_1} = -\frac{p_1}{p_2}.$$

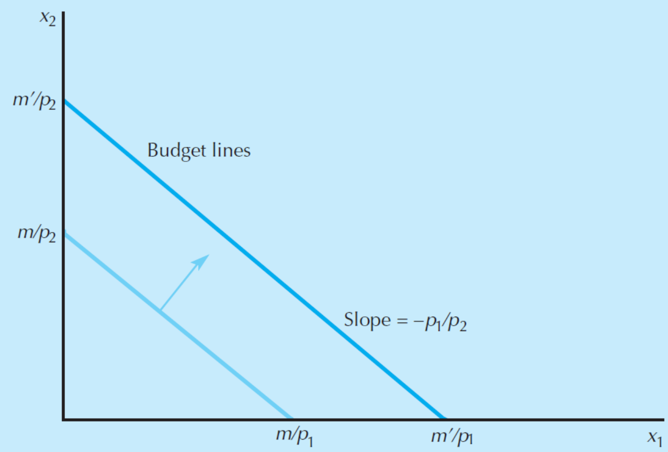
This is just the slope of the budget line. The negative sign is there since $\mathrm{d}x_1$ and $\mathrm{d}x_2$ must always have opposite signs. If you consume more of good 1, you have to consume less of good 2 and vice versa if you continue to satisfy the budget constraint.

The slope of the budget line measures the **opportunity cost** of consuming good 1. In order to consume more of good 1 you have to give up some consumption of good 2.

2.3 How the Budget Line Changes

When prices and incomes change, the set of goods that a consumer can afford changes as well.

Let us first consider changes in income. It is easy to see from equation (2.4) that an increase in income will increase the vertical intercept and not affect the slope of the line. Thus an increase in income will result in a parallel shift outward of the budget line as in Figure 2.2. Similarly, a decrease in income will cause a parallel shift inward.



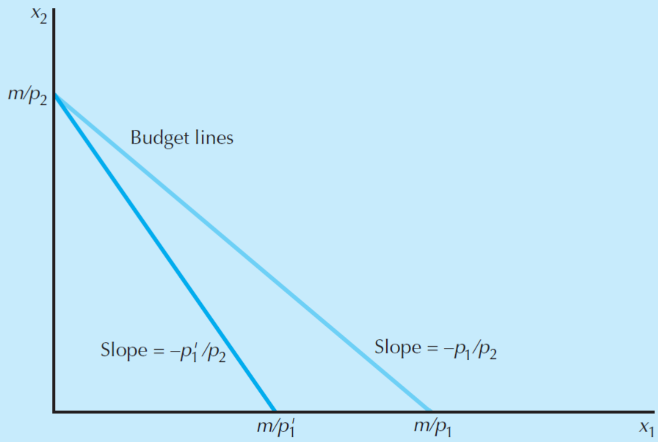
Increasing income. An increase in income causes a parallel shift outward of the budget line.

Figure 2.2: Increasing income

What about changes in prices? Let us first consider increasing price 1 while holding price 2 and income fixed. According to equation (2.4), increasing p_1 will not change the vertical intercept, but it will make the budget line steeper since p_1/p_2 will become larger. What happens to the budget line when we change the prices of good 1 and good 2 at the same time? Suppose for example that we double the prices of both goods 1 and 2. In this case both the horizontal and vertical intercepts shift inward by a factor of one-half, and therefore the budget line shifts inward by one-half as well. Multiplying both prices by two is just like dividing income by 2.

We can also see this algebraically. Suppose our original budget line is

$$p_1x_1 + p_2x_2 = m.$$



Increasing price. If good 1 becomes more expensive, the budget line becomes steeper.

Figure 2.3: Increasing price

Now suppose that both prices become t times as large. Multiplying both prices by t yields

$$tp_1x_1 + tp_2x_2 = m.$$

But this equation is the same as

$$p_1x_1 + p_2x_2 = \frac{m}{t}.$$

Thus multiplying both prices by a constant amount t is just like dividing income by the same constant t. It follows that if we multiply both prices by t and we multiply income by t, then the budget line won't change at all.

We can also consider price and income changes together.

✎ **Remark:2.2 The Numeraire** ▽

The budget line is defined by two prices and one income, but one of these variables is redundant. We could peg one of the prices, or the income, to some fixed value, and adjust the other variables so as to describe exactly the same budget set. Thus the budget line

$$p_1x_1 + p_2x_2 = m$$

is exactly the same budget line as

$$\frac{p_1}{p_2}x_1 + x_2 = \frac{m}{p_2}$$

or

$$\frac{p_1}{m}x_1 + \frac{p_2}{m}x_2 = 1.$$

Pegging the price of one of the goods or income to 1 and adjusting the other price and income appropriately doesn't change the budget set at all.
When we set one of the prices to 1, as we did above, we often refer to that price as the **numeraire** price. The numeraire price is the price relative to which we are measuring the other price and income.

2.4 Taxes, Subsidies and Rationing

Economic policy often uses tools that affect a consumer's budget constraint, such as taxes. For example, if the government imposes a **quantity tax**, this means that the consumer has to pay a certain amount to the government for each unit of the good he purchases. From the viewpoint of the consumer the tax is just like a higher price. Thus a quantity tax of t dollars per unit of good 1 simply changes the price of good 1 from p_1 to $p_1 + t$.

Another kind of tax is a **value tax**. As the name implies this is a tax on the value—the price—of a good, rather than the quantity purchased of a good. A value tax is usually expressed in percentage terms.

A **subsidy** is the opposite of a tax. In the case of a **quantity subsidy**, the government gives an amount to the consumer that depends on the amount of the good purchased. Similarly an **ad valorem subsidy** is a subsidy based on the price of the good being subsidized.

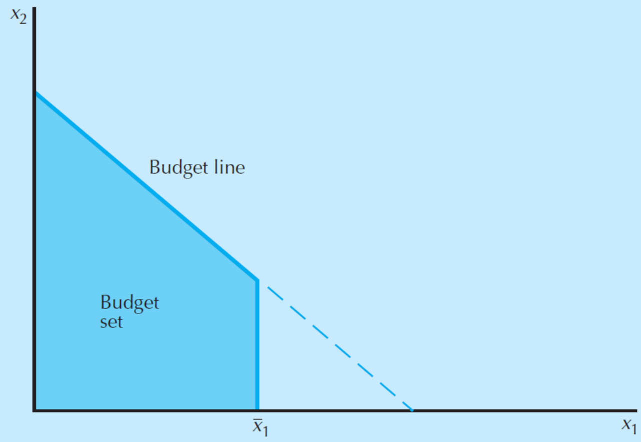
✎ Remark:2.3 ▽

You can see that taxes and subsidies affect prices in exactly the same way except for the algebraic sign: a tax increases the price to the consumer, and a subsidy decreases it.

Another kind of tax or subsidy that the government might use is a **lumpsum tax** or **subsidy**. In the case of a tax, this means that the government takes away some fixed amount of money, regardless of the individual's behavior. Thus a lump-sum tax means that the budget line of a consumer will shift inward because his money income has been reduced. Similarly, a lump-sum subsidy means that the budget line will shift outward.

Governments also sometimes impose **rationing** constraints. This means that the level of consumption of some good is fixed to be no larger than some amount. Suppose, for example, that good 1 were rationed so that no more than $\bar{x_1}$ could be consumed by a given consumer. Then the budget set of the consumer would look like that depicted in Figure 2.4: it would be the old budget set with a piece lopped off. The lopped-off piece consists of all the consumption bundles that are affordable but have $x_1 > \bar{x_1}$.

Sometimes taxes, subsidies, and rationing are combined. For example, we could consider a situation where a consumer could consume good 1 at a price of p_1 up to some level $\bar{x_1}$, and then had to pay a tax t on all consumption in excess of $\bar{x_1}$. The budget set for this consumer



Budget set with rationing. If good 1 is rationed, the section of the budget set beyond the rationed quantity will be lopped off.

Figure 2.4: Budget Set with Rationing

is depicted in Figure 2.5. Here the budget line has a slope of $-p_1/p_2$ to the left of $\bar{x_1}$, and a slope of $-(p_1+t)/p_2$ to the right of $\bar{x_1}$.

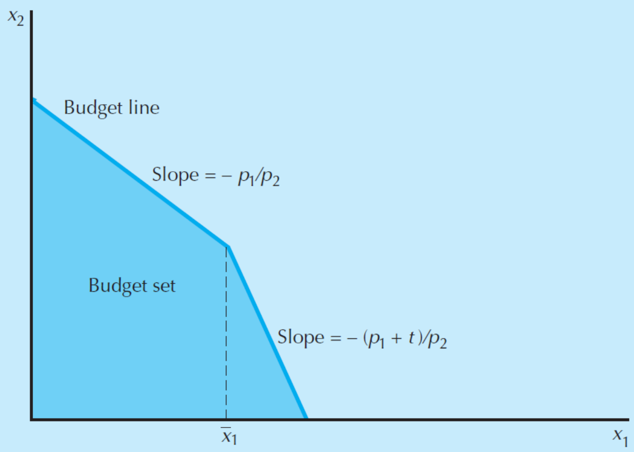
The Food Stamp Program

The pre-1979 Food Stamp program was an ad valorem subsidy on food. The rate at which food was subsidized depended on the household income. In 1979 the Food Stamp program was modified. Instead of requiring that households purchase food stamps, they are now simply given to qualified households. The Food Stamp program is effectively a lump-sum subsidy, except for the fact that the food stamps can't be sold.

✎ **Remark:2.4** ▽

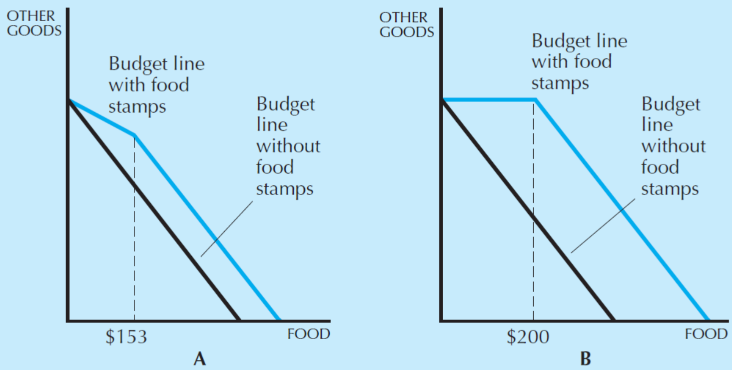
First, we can observe that since the budget set doesn't change when we multiply all prices and income by a positive number, the optimal choice of the consumer from the budget set can't change either. Without even analyzing the choice process itself, we have derived an important conclusion: *a perfectly balanced inflation—one in which all prices and all incomes rise at the same rate—doesn't change anybody's budget set, and thus cannot change anybody's optimal choice.*

Second, suppose that the consumer's income increases and all prices remain the same. This represents a parallel shift outward of the budget line. Thus every bundle the



Taxing consumption greater than $\overline{x}_1$. In this budget set the consumer must pay a tax only on the consumption of good 1 that is in excess of $\overline{x}_1$, so the budget line becomes steeper to the right of $\overline{x}_1$.

Figure 2.5: Tax Consumption



Food stamps. How the budget line is affected by the Food Stamp program. Part A shows the pre-1979 program and part B the post-1979 program.

Figure 2.6: Food Stamps

consumer was consuming at the lower income is also a possible choice at the higher income. Then the consumer must be at least as well-off at the higher income as at the lower income—since he or she has the same choices available as before plus some more. Similarly, if one price declines and all others stay the same, the consumer must be at least as well-off.

3 Preferences

The last chapter was devoted to clarifying the meaning of “can afford”, and this chapter will be devoted to clarifying the economic concept of “best things”.

We call the objects of consumer choice **consumption bundles**. This is a complete list of the goods and services that are involved in the choice problem that we are investigating.

Let us take our consumption bundle to consist of two goods, and let x_1 denote the amount of one good and x_2 the amount of the other. The complete consumption bundle is therefore denoted by (x_1, x_2). As noted before, we will occasionally abbreviate this consumption bundle by X.

3.1 Consumer Preferences

We will suppose that given any two consumption bundles, (x_1, x_2) and (y_1, y_2), the consumer can rank them as to their desirability. That is, the consumer can determine that one of the consumption bundles is strictly better than the other, or decide that she is indifferent between the two bundles. We will use the symbol $\succ$ to mean that one bundle is **strictly preferred** to another, so that $(x_1, x_2) \succ (y_1, y_2)$ should be interpreted as saying that the consumer strictly prefers (x_1, x_2) to (y_1, y_2), in the sense that she definitely wants the X-bundle rather than the Y-bundle. This preference relation is meant to be an operational notion. Thus the idea of preference is based on the consumer's behavior.

If the consumer is **indifferent** between two bundles of goods, we use the symbol $\sim$ and write $(x_1, x_2) \sim (y_1, y_2)$. Indifference means that the consumer would be just as satisfied, according to her own preferences, consuming the bundle (x_1, x_2) as she would be consuming the other bundle, (y_1, y_2). If the consumer prefers or is indifferent between the two bundles we say that he **weakly prefers** (x_1, x_2) to (y_1, y_2) and write $(x_1, x_2) \succeq (y_1, y_2)$.

3.2 Assumptions about Preferences

We usually make assumptions about how the preference relations work. Some of the assumptions are so fundamental that we can refer to them as axioms of consumer theory.

❏ **Axiom:3.1 Complete** ▽

We assume that any two bundles can be compared. That is, given any X-bundle and any Y-bundle, we assume that $(x_1, x_2) \succeq (y_1, y_2)$, or $(y_1, y_2) \succeq (x_1, x_2)$, or both, in which case the consumer is indifferent between the two bundles.

❏ **Axiom:3.2 Reflexive** ▽

We assume that any bundle is at least as good as itself: $(x_1, x_2) \succeq (x_1, x_2)$.

❏ Axiom:3.3 Transitive ▽

If $(x_1, x_2) \succeq (y_1, y_2)$ and $(y_1, y_2) \succeq (z_1, z_2)$, then we assume that $(x_1, x_2) \succeq (z_1, z_2)$. In other words, if the consumer thinks that X is at least as good as Y and that Y is at least as good as Z, then the consumer thinks that X is at least as good as Z.

❏ Axiom:3.4 Antisymmetrical ▽

If $(x_1, x_2) \succeq (y_1, y_2)$ and $(y_1, y_2) \succeq (x_1, x_2)$ we can conclude that $(x_1, x_2) \sim (y_1, y_2)$.

✎ Remark:3.5 ▽

In the Discrete Mathematics course, we know that if a relation on a set R is reflexive, transitive, and antisymmetrical, then it is a **partial order** relation on the set R. In particular, if it is also complete, it is a **full order** relation on the set R.
So that the preference relation is a **full order** on the goods set.

3.3 Indifference Curves

Consider Figure 3.1 where we have illustrated two axes representing a consumer's consumption of goods 1 and 2. Let us pick a certain consumption bundle (x_1, x_2) and shade in all of the consumption bundles that are weakly preferred to (x_1, x_2). This is called the **weakly preferred set**. The bundles on the boundary of this set—the bundles for which the consumer is just indifferent to (x_1, x_2)—form the **indifference curve**.

❖ Definition:3.6 Indifference Curve ▽

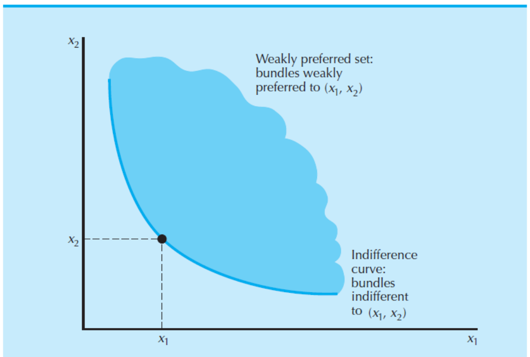
We can draw an indifference curve through any consumption bundle we want. The indifference curve through a consumption bundle consists of all bundles of goods that leave the consumer indifferent to the given bundle.

We can state an important principle about indifference curves: indifference curves representing distinct levels of preference cannot cross. That is, the situation depicted in Figure 3.2 cannot occur.

Perfect Substitutes

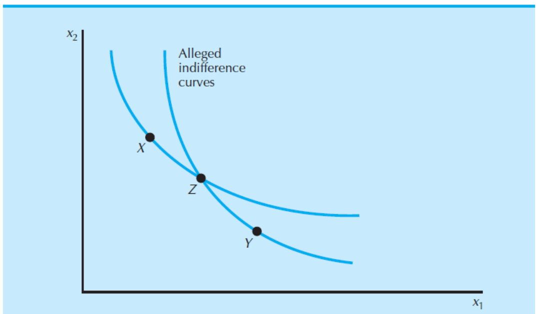
Two goods are **perfect substitutes** if the consumer is willing to substitute one good for the other at a constant rate. The simplest case of perfect substitutes occurs when the consumer is willing to substitute the goods on a one-to-one basis.

The important fact about perfect substitutes is that the indifference curves have a constant slope.



Weakly preferred set. The shaded area consists of all bundles that are at least as good as the bundle (x_1, x_2).

Figure 3.1: Weekly Preferred Set



Indifference curves cannot cross. If they did, X, Y, and Z would all have to be indifferent to each other and thus could not lie on distinct indifference curves.

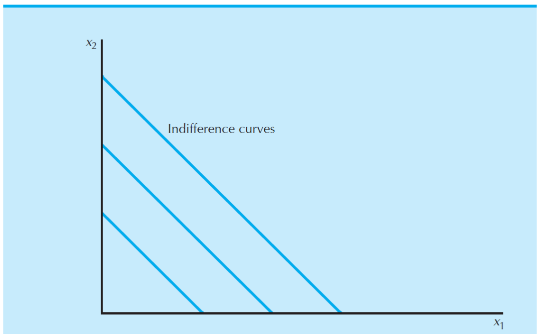
Figure 3.2: Indifference Curve Cannot Cross

Perfect Complements

Perfect complements are goods that are always consumed together in fixed proportions. In some sense the goods "complement" each other.

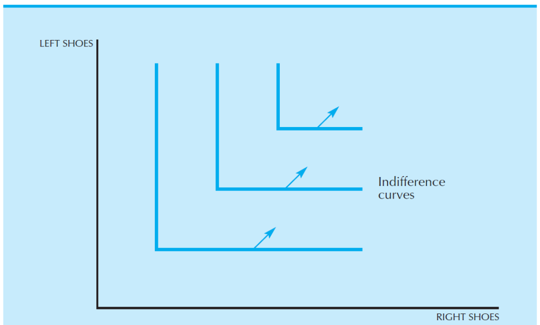
Thus the indifference curves are L-shaped, with the vertex of the L occurring where the number of left shoes equals the number of right shoes as in Figure 3.4.

The important thing about perfect complements is that the consumer prefers to consume the goods in fixed proportions, not necessarily that the proportion is one-to-one.



Perfect substitutes. The consumer only cares about the total number of pencils, not about their colors. Thus the indifference curves are straight lines with a slope of −1.

Figure 3.3: Perfect Substitutes



Perfect complements. The consumer always wants to consume the goods in fixed proportions to each other. Thus the indifference curves are L-shaped.

Figure 3.4: Perfect Complements

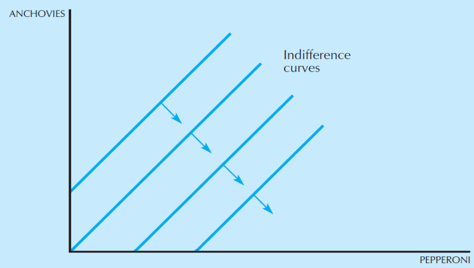
Bads

A **bad** is a commodity that the consumer doesn't like. Pick a bundle (x_1, x_2) consisting of some pepperoni and some anchovies. If we give the consumer more anchovies, what do we have to do with the pepperoni to keep him on the same indifference curve? Clearly, we have to give him some extra pepperoni to compensate him for having to put up with the anchovies. Thus this consumer must have indifference curves that slope up and to the right as depicted in Figure 3.5.

The direction of increasing preference is down and to the right—that is, toward the direction of decreased anchovy consumption and increased pepperoni consumption, just as the arrows in the diagram illustrate.

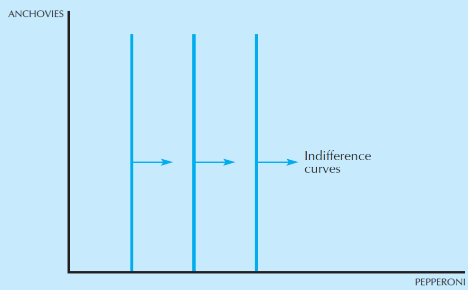
Neutrals

A good is a **neutral good** if the consumer doesn't care about it one way or the other. In this case his indifference curves will be vertical lines as depicted in Figure 3.6.



Bads. Here anchovies are a "bad," and pepperoni is a "good" for this consumer. Thus the indifference curves have a positive slope.

Figure 3.5: Bads



A neutral good. The consumer likes pepperoni but is neutral about anchovies, so the indifference curves are vertical lines.

Figure 3.6: A Neutral Good

Satiation

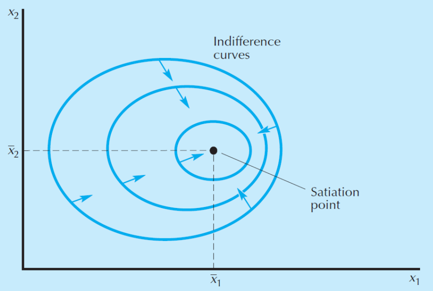
We sometimes want to consider a situation involving **satiation**, where there is some overall best bundle for the consumer, and the "closer" he is to that best bundle, the better off he is in terms of his own preferences. For example, suppose that the consumer has some most preferred bundle of goods $(\bar{x_1}, \bar{x_2})$, and the farther away he is from that bundle, the worse off he is. In this case we say that $(\bar{x_1}, \bar{x_2})$ is a **satiation point**, or a **bliss point**. The indifference curves for the consumer look like those depicted in Figure 3.7. The best point is $(\bar{x_1}, \bar{x_2})$ and points farther away from this bliss point lie on "lower" indifference curves.

In this case the indifference curves have a negative slope when the consumer has "too little" or "too much" of both goods, and a positive slope when he has "too much" of one of the goods. When he has too much of one of the goods, it becomes a bad—reducing the consumption of the bad good moves him closer to his "bliss point". If he has too much of both goods, they both are bads, so reducing the consumption of each moves him closer to the bliss point.

Discrete Goods

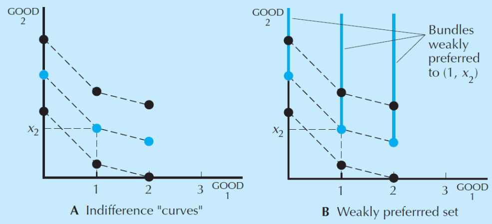
Sometimes we want to examine preferences over goods that naturally come in discrete units.

There is no difficulty in using preferences to describe choice behavior for this kind of **discrete good**. Suppose that x_2 is money to be spent on other goods and x_1 is a discrete good that is only available in integer amounts. We have illustrated the appearance of indifference "curves" and a weakly preferred set for this kind of good in Figure 3.8. In this case the bundles indifferent to a given bundle will be a set of discrete points. The set of bundles at least as good as a particular bundle will be a set of line segments.



Satiated preferences. The bundle $(\bar{x}_1, \bar{x}_2)$ is the satiation point or bliss point, and the indifference curves surround this point.

Figure 3.7: Satiated Preferences



A discrete good. Here good 1 is only available in integer amounts. In panel A the dashed lines connect together the bundles that are indifferent, and in panel B the vertical lines represent bundles that are at least as good as the indicated bundle.

Figure 3.8: A Discrete Good

3.4 Well-Behaved Preferences

In this section we will describe some more general assumptions that we will typically make about preferences and the implications of these assumptions for the shapes of the associated indifference curves. These assumptions are not the only possible ones; in some situations you might want to use different assumptions. But we will take them as the defining features for **well-behaved indifference curves**.

❖ Definition:3.7 Well-behaved Preferences ▽

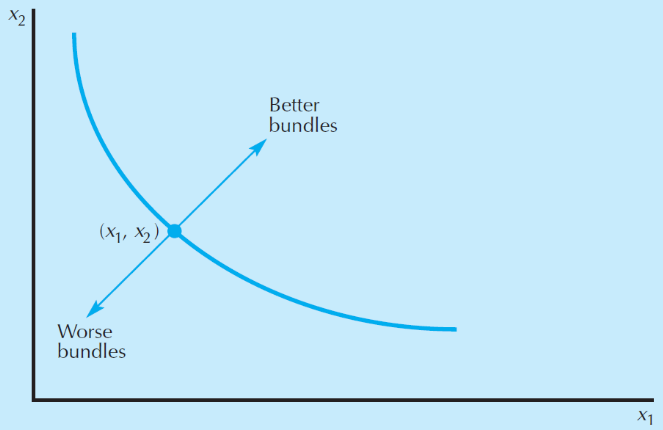
Well-behaved preferences are **monotonic** (meaning more is better) and **convex** (meaning averages are preferred to extremes).

First we will typically assume that we are talking about goods, not bads. More precisely, if (x_1, x_2) is a bundle of goods and (y_1, y_2) is a bundle of goods with at least as much of both goods and more of one, then $(y_1, y_2) \succ (x_1, x_2)$. This assumption is sometimes called

monotonicity of preferences. The assumption of monotonicity is saying only that we are going to examine situations before satiation point is reached, while more still is better.

Monotonicity imply about the shape of indifference curves have a negative slope. Second, we are going to assume that averages are preferred to extremes. Actually, we're going to assume this for any weight t between 0 and 1. Thus we are assuming that if $(x_1, x_2) \sim (y_1, y_2)$, then

$$(tx_1 + (1-t)y_1, tx_2 + (1-t)y_2) \succeq (x_1, x_2).$$

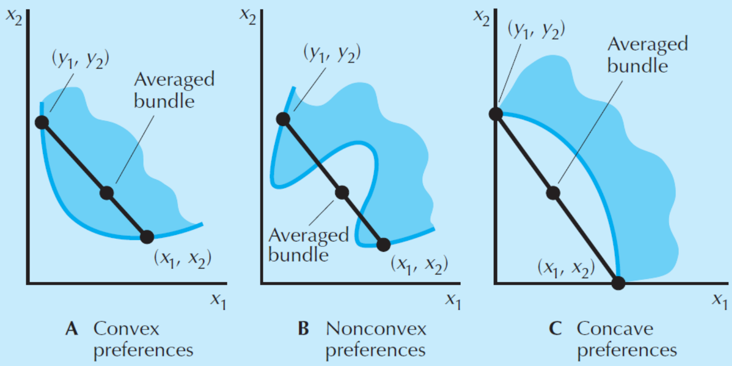


Monotonic preferences. More of both goods is a better bundle for this consumer; less of both goods represents a worse bundle.

Figure 3.9: Monotonic Preferences

This assumption about preferences mean geometrically that the set of bundles weakly preferred to (x_1, x_2) is a **convex set**. For suppose that (y_1, y_2) and (x_1, x_2) are indifferent bundles. Then, if averages are preferred to extremes, all of the weighted averages of (x_1, x_2) and (y_1, y_2) are weakly preferred to (x_1, x_2) and (y_1, y_2). A convex set has the property that if you take any two points in the set and draw the line segment connecting those two points, that line segment lies entirely in the set. Figure 3.10A depicts an example of convex preferences, while Figures 3.10B and 3.10C show two examples of nonconvex preferences. Figure 3.10C presents preferences that we call them "concave preferences".

Why do we want to assume that well-behaved preferences are convex? Because, for the most part, goods are consumed together. The kinds of preferences depicted in Figures 3.10B and 3.10C imply that the consumer would prefer to specialize, at least to some degree, and to consume only one of the goods. However, the normal case is where the consumer would want to trade some of one good for the other and end up consuming some of each, rather than



Various kinds of preferences. Panel A depicts convex preferences, panel B depicts nonconvex preferences, and panel C depicts "concave" preferences.

Figure 3.10: Various Kinds of Preferences

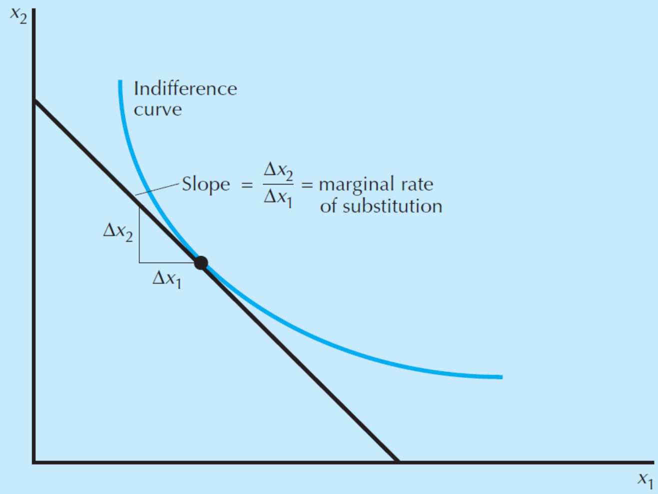
specializing in consuming only one of the two goods. Finally, one extension of the assumption of convexity is the assumption of **strict convexity**. This means that the weighted average of two indifferent bundles is strictly preferred to the two extreme bundles. Convex preferences may have flat spots, while strictly convex preferences must have indifferences curves that are "rounded." The preferences for two goods that are perfect substitutes are convex, but not strictly convex.

3.5 The Marginal Rate of Substitution

❖ Definition:3.8 MRS ▽

The slope of an indifference curve is known as the marginal rate of substitution (MRS). The name comes from the fact that the MRS measures the rate at which the consumer is just willing to substitute one good for the other.

The Marginal Rate of Substitution (MRS) quantifies the rate at which a consumer compensates for a marginal loss of good 1 Δx_1 by gaining good 2 Δx_2 to remain on the same indifference curve, defined as $|\text{MRS}| = \mathrm{d}x_2/\mathrm{d}x_1$, the absolute slope of the indifference curve. As Δx_1 becomes infinitesimally small $\mathrm{d}x_1$, this ratio represents the slope of the curve, inherently negative due to **monotonic** preferences. For a consumer with well-behaved preferences (monotonic and convex) at bundle (x_1, x_2), an exchange rate E defines feasible trades along a line with slope $-E$: moving leftward (trading good 1 for 2) or downward (trading good 2 for 1) preserves the trade-off ratio, where E corresponds to the rate of substitution between goods



The marginal rate of substitution (MRS). The marginal rate of substitution measures the slope of the indifference curve.

Figure 3.11: The MRS

while maintaining utility.

We note that any time the exchange line crosses the indifference curve, there will be some points on that line that are preferred to (x_1, x_2)—that lie above the indifference curve. Thus, if there is to be no movement from (x_1, x_2), the exchange line must be tangent to the indifference curve, the slope of the exchange line, $-E$, must be the slope of the indifference curve at (x_1, x_2). At any other rate of exchange, the exchange line would cut the indifference curve and thus allow the consumer to move to a more preferred point.

✎ **Remark:3.9** ▽

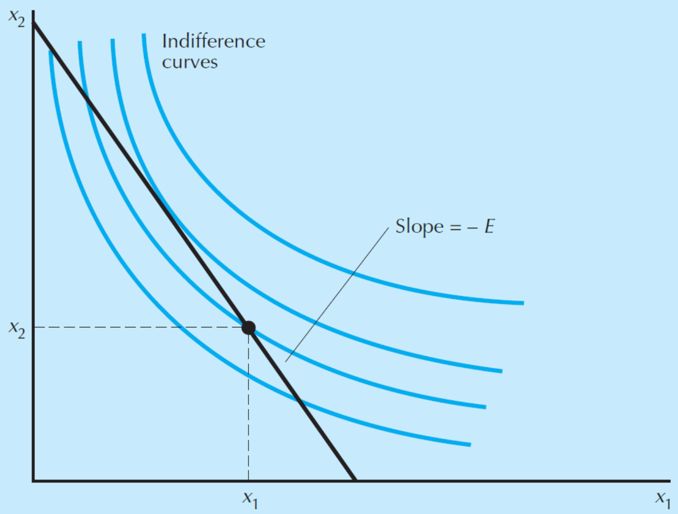
How much you actually end up buying of a good will depend on your preferences for that good and the prices that you face. How much you would be willing to pay for a small amount extra of the good is a feature only of your preferences.

♣ **Corollary:3.10 Behavior of the MRS** ▽

The **perfect substitutes** indifference curves are characterized by the fact that the MRS is constant at -1.

The **neutrals** case is characterized by the fact that the MRS is everywhere infinite.

The preferences for **perfect complements** are characterized by the fact that the MRS is either zero or infinity, and nothing in between.



Trading at an exchange rate. Here we are allowing the consumer to trade the goods at an exchange rate E, which implies that the consumer can move along a line with slope $-E$.

Figure 3.12: Trading at An Exchange Rate

The assumption of **monotonicity** implies that indifference curves must have a negative slope.
For **strictly convex** indifference curves, the MRS decreases (in absolute value) as we increase x_1. Thus the indifference curves exhibit a diminishing marginal rate of substitution.

4 Utility

The theory of consumer behavior has been reformulated entirely in terms of **consumer preferences**, and utility is seen only as a way to describe preferences.

A **utility function** is a way of assigning a number to every possible consumption bundle such that more-preferred bundles get assigned larger numbers than less-preferred bundles. That is, a bundle (x_1, x_2) is preferred to a bundle (y_1, y_2) if and only if the utility of (x_1, x_2) is larger than the utility of (y_1, y_2): in symbols, $(x_1, x_2) \succ (y_1, y_2)$ if and only if $u(x_1, x_2) > u(y_1, y_2)$.

The only property of a utility assignment that is important is how it orders the bundles of goods. The magnitude of the utility function is only important insofar as it ranks the different consumption bundles; the size of the utility difference between any two consumption bundles doesn't matter. Because of this emphasis on ordering bundles of goods, this kind of utility is referred to as **ordinal utility**.

❖ Definition:4.1 Monotonic Transformation ▽

A **monotonic transformation** is a way of transforming one set of numbers into another set of numbers in a way that preserves the order of the numbers.
We represent a monotonic transformation by a function $f(u)$, in a way that preserves the order of the numbers in the sense that $u_1 > u_2$ implies $f(u_1) > f(u_2)$. A monotonic transformation and a monotonic function are essentially the same thing.

Provided that a monotonic function $f(u)$ is differentiable, its derivative $f'(u)$ is given by

$$f'(u) = \lim_{\hat{u} \to u} \frac{f(\hat{u}) - f(u)}{\hat{u} - u} > 0.$$

For a monotonic transformation, $f(\hat{u}) - f(u)$ always has the same sign as $\hat{u} - u$. Thus amonotonic function always has a positive first derivative. This means that the graph of a monotonic function will always have a positive slope.

❝ Proposition:4.2 ▽

If $f(u)$ is any monotonic transformation of a utility function that represents some particular preferences, then $f(u(x_1, x_2))$ is also a utility function that represents those same preferences.

Proof: 1. To say that $u(x_1, x_2)$ represents some particular preferences means that $u(x_1, x_2) > u(y_1, y_2)$ if and only if $(x_1, x_2) \succ (y_1, y_2)$.

2. If $f(u)$ is a monotonic transformation, then $u(x_1, x_2) > u(y_1, y_2)$ if and only if $f(u(x_1, x_2)) > f(u(y_1, y_2))$.

3. $f(u(x_1, x_2)) > f(u(y_1, y_2))$ if and only if $(x_1, x_2) \succ (y_1, y_2)$

So the function $f(u)$ represents the preferences in the same way as the original utility function $u(x_1, x_2)$. □

4.1 Some Examples of Utility Functions

Perfect Substitutes

In general, preferences for perfect substitutes can be represented by a utility function of the form

$$u(x_1, x_2) = ax_1 + bx_2.$$

Here a and b are some positive numbers that measure the "value" of goods 1 and 2 to the consumer. Note that the slope of a typical indifference curve is given by $-a/b$.

Perfect Complements

In general, a utility function that describes perfect-complement preferences is given by

$$u(x_1, x_2) = \min\{ax_1, bx_2\},$$

where a and b are positive numbers that indicate the proportions in which the goods are consumed.

Quasilinear Preferences

Suppose that a consumer has indifference curves that are vertical translates of one another, as in Figure 4.1. This means that all of the indifference curves are just vertically "shifted" versions of one indifference curve. It follows that the equation for an indifference curve takes the form $x_2 = k - v(x_1)$, where k is a different constant for each indifference curve.

The natural way to label indifference curves here is with k—roughly speaking, the height of the indifference curve along the vertical axis. Solving for k and setting it equal to utility, we have

$$u(x_1, x_2) = k = v(x_1) + x_2.$$

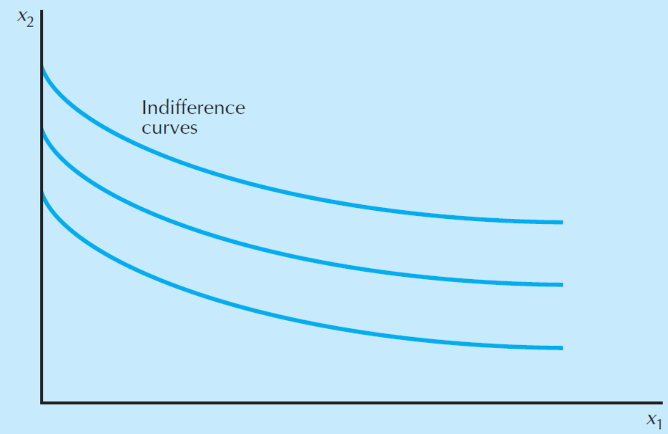
In this case the utility function is linear in good 2, but (possibly) nonlinear in good 1; hence the name **quasilinear utility**, meaning "partly linear" utility.

Cobb-Douglas Preferences

Another commonly used utility function is the Cobb-Douglas utility function

$$u(x_1, x_2) = x_1^c x_2^d,$$

where c and d are positive numbers that describe the preferences of the consumer.



Quasilinear preferences. Each indifference curve is a vertically shifted version of a single indifference curve.

Figure 4.1: Quasilinear Perferences

Cobb-Douglas preferences are the standard example of indifference curves that look well-behaved, and in fact the formula describing them is about the simplest algebraic expression that generates well-behaved preferences.

Suppose that we start with the Cobb-Douglas form

$$u(x_1, x_2) = x_1^c x_2^d.$$

Then raising utility to the $1/(c + d)$ power, we have

$$x_1^{\frac{c}{c+d}} x_2^{\frac{d}{c+d}}.$$

Now define a new number

$$a = \frac{c}{c + d}.$$

We can now write our utility function as

$$v(x_1, x_2) = x_1^a x_2^{1-a}.$$

This means that we can always take a monotonic transformation of the Cobb-Douglas utility function that make the exponents sum to 1.

4.2 Marginal Utility

Consider a consumer who is consuming some bundle of goods, (x_1, x_2). This consumer's utility change as we give him or her a little more of good 1, the rate of change is called the **marginal utility** with respect to good 1. We write it as MU_1 and think of it as being a ratio that measures the rate of change in utility associated with a small change in the amount of good 1, holding the amount of good 2 fixed.

❖ Definition:4.3 Marginal Utility ▽

As elsewhere in economics, “marginal” just means a derivative. So the marginal utility of good 1 is just

$$MU_1 = \lim_{\Delta x_1 \to 0} \frac{u(x_1 + \Delta x_1, x_2) - u(x_1, x_2)}{\Delta x_1} = \frac{\partial u(x_1, x_2)}{\partial x_1}.$$

This definition implies that to calculate the change in utility associated with a small change in consumption of good 1, we can just multiply the change in consumption by the marginal utility of the good:

$$\mathrm{d}U = MU_1 \mathrm{d}x_1.$$

A utility function $u(x_1, x_2)$ can be used to measure the marginal rate of substitution (MRS). The MRS measures the slope of the indifference curve at a given bundle of goods; it can be interpreted as the rate at which a consumer is just willing to substitute a small amount of good 2 for good 1.

We can formally derive the MRS by using differentials, For this method, we consider making a change $(\mathrm{d}x_1, \mathrm{d}x_2)$ that keeps utility constant. So we want

$$\mathrm{d}u = \frac{\partial u(x_1, x_2)}{\partial x_1}\mathrm{d}x_1 + \frac{\partial u(x_1, x_2)}{\partial x_2}\mathrm{d}x_2 = 0.$$

This term measures the increase in utility from the small change $\mathrm{d}x_1$, and the second term measures the increase in utility from the small change $\mathrm{d}x_2$. We want to pick these changes so that the total change in utility, $\mathrm{d}u$, is zero. Solving for $\mathrm{d}x_2/\mathrm{d}x_1$ gives us

$$\mathrm{MRS} = \frac{\mathrm{d}x_2}{\mathrm{d}x_1} = -\frac{\partial u(x_1, x_2)/\partial x_1}{\partial u(x_1, x_2)/\partial x_2} = -\frac{MU_1}{MU_2}.$$

Cobb-Douglas Preferences

The MRS for Cobb-Douglas preferences is easy to calculate by using the formula derived above.

If we choose the log representation where

$$u(x_1, x_2) = c \ln x_1 + d \ln x_2,$$

then we have

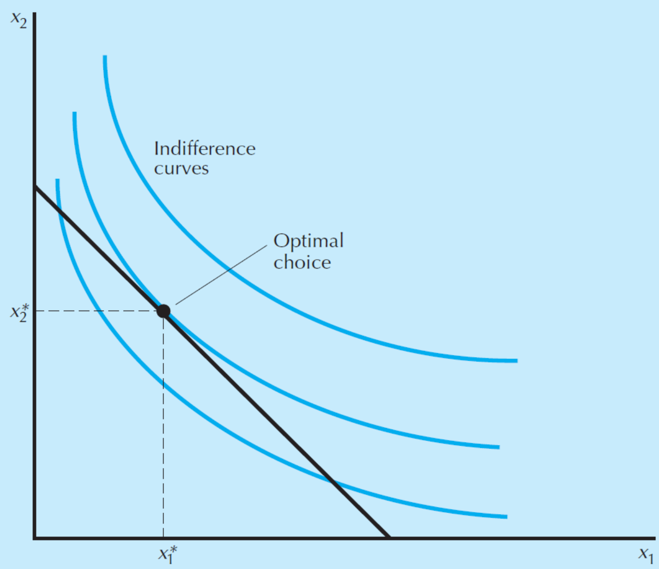
$$\begin{aligned} \mathrm{MRS} &= -\frac{\partial u(x_1, x_2)/\partial x_1}{\partial u(x_1, x_2)/\partial x_2} \\ &= -\frac{c/x_1}{d/x_2} \\ &= -\frac{cx_2}{dx_1}. \end{aligned}$$

Note that the MRS only depends on the ratio of the two parameters and the quantity of the two goods in this case.

5 Choice

In this chapter we will put together the budget set and the theory of preferences in order to examine the **optimal choice** of consumers. The economic model of consumer choice is that consumers choose the most preferred bundle from their budget sets.

5.1 Optimal Choice



Optimal choice. The optimal consumption position is where the indifference curve is tangent to the budget line.

Figure 5.1: Optimal Choice

In the diagram, the bundle of goods that is associated with the highest indifference curve that just touches the budget line is labeled (x_1^*, x_2^*). The choice (x_1^*, x_2^*) is an **optimal choice** for the consumer.

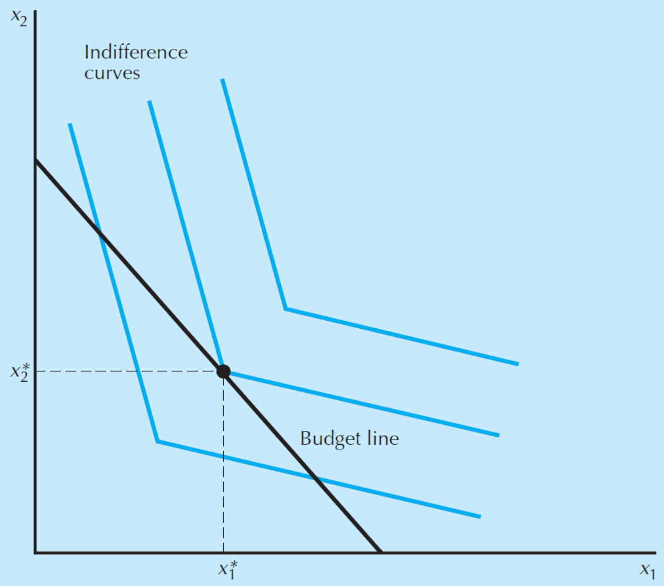
✎ **Remark:5.1** ▽

Note an important feature of this optimal bundle: at this choice, the indifference curve is tangent to the budget line.

What is always true is that at the optimal point the indifference curve can't cross the budget line. So when does "not crossing" imply tangent? Let's look at the exceptions first.

First, the indifference curve might not have a tangent line, as in Figure 5.2. Here the indifference curve has a kink at the optimal choice, and a tangent just isn't defined, since the mathematical definition of a tangent requires that there be a unique tangent line at each point.

The second exception is more interesting. Suppose that the optimal point occurs where the consumption of some good is zero as in Figure 5.3. Then the slope of the indifference curve and the slope of the budget line are different, but the indifference curve still doesn't cross the budget line. We say that Figure 5.3 represents a **boundary optimum**, while a case like Figure 5.1 represents an **interior optimum**.



Kinky tastes. Here is an optimal consumption bundle where the indifference curve doesn't have a tangent.

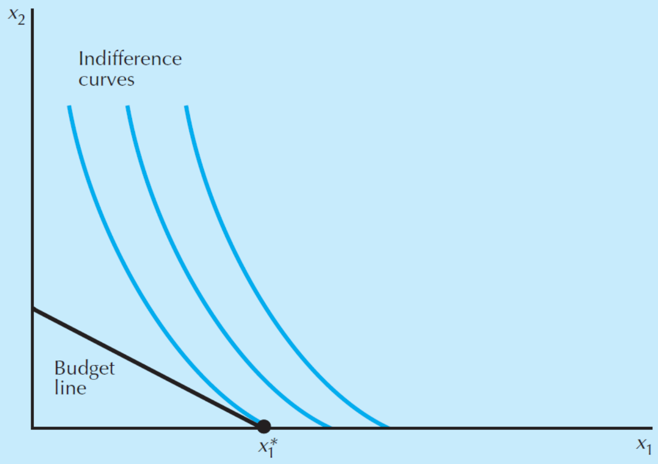
Figure 5.2: Kinkt Tastes

We've found a necessary condition that the optimal choice must satisfy. If the optimal choice involves consuming some of both goods—so that it is an interior optimum—then necessarily the indifference curve will be tangent to the budget line.

Look at Figure 5.4. Here we have three bundles where the tangency condition is satisfied, all of them interior, but only two of them are optimal. So in general, the tangency condition is only a necessary condition for optimality, not a sufficient condition.

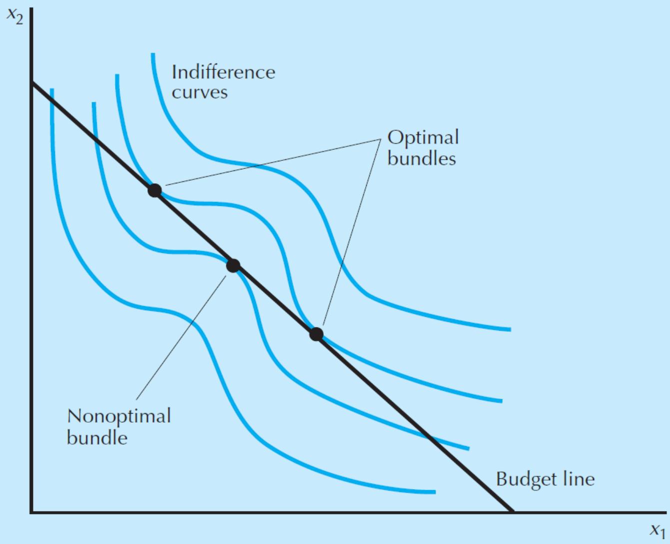
However, there is one important case where it is sufficient: the case of convex preferences.

Figure 5.4 also shows us that in general there may be more than one optimal bundle that satisfies the tangency condition. However, again convexity implies a restriction. If the



Boundary optimum. The optimal consumption involves consuming zero units of good 2. The indifference curve is not tangent to the budget line.

Figure 5.3: Boundary Optimum



More than one tangency. Here there are three tangencies, but only two optimal points, so the tangency condition is necessary but not sufficient.

Figure 5.4: More Than One Tangency

indifference curves are strictly convex—they don't have any flat spots—then there will be only one optimal choice on each budget line.

The condition that the MRS must equal the slope of the budget line at an interior optimum is obvious graphically.

If the consumer is at a consumption bundle where he or she is willing to stay put, it must be one where the MRS is equal to this rate of exchange:

$$\text{MRS} = -\frac{p_1}{p_2}.$$

Whenever the MRS is different from the price ratio, the consumer cannot be at his or her optimal choice.

5.2 Consumer Demand

The optimal choice of goods 1 and 2 at some set of prices and income is called the consumer's **demanded bundle**. In general when prices and income change, the consumer's optimal choice will change. The **demand function** is the function that relates the optimal choice—the quantities demanded—to the different values of prices and incomes.

We will write the demand functions as depending on both prices and income: $x_1(p_1, p_2, m)$ and $x_2(p_1, p_2, m)$. For each different set of prices and income, there will be a different combination of goods that is the optimal choice of the consumer.

Perfect Substitutes

The case of perfect substitutes is illustrated in Figure 5.5. All they say is that if two goods are perfect substitutes, then a consumer will purchase the cheaper one. If both goods have the same price, then the consumer doesn't care which one he or she purchases. The demand function for good 1 will be

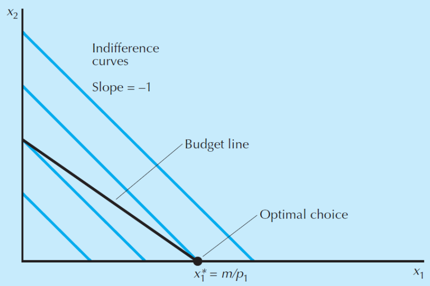
$$x_1 = \begin{cases} m/p_1 & \text{when } p_1 < p_2 \\ \text{any number between 0 and } m/p_1 & \text{when } p_1 = p_2 \\ 0 & \text{when } p_1 > p_2 \end{cases}$$

Perfect Complements

The case of perfect complements is illustrated in Figure 5.6. Note that the optimal choice must always lie on the diagonal, where the consumer is purchasing equal amounts of both goods, no matter what the prices are. Let this amount be denoted by x. Then we have to satisfy the budget constraint

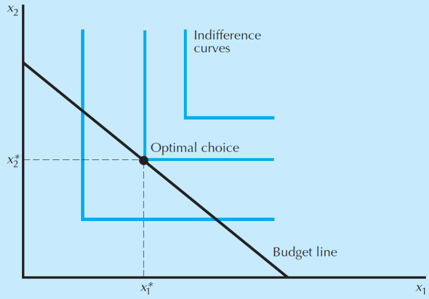
$$p_1 x_x + p_2 x_2 = m.$$

Solving for x gives us the optimal choices of goods 1 and 2:



Optimal choice with perfect substitutes. If the goods are perfect substitutes, the optimal choice will usually be on the boundary.

Figure 5.5: Perfect preferences



Optimal choice with perfect complements. If the goods are perfect complements, the quantities demanded will always lie on the diagonal since the optimal choice occurs where x_1 equals x_2.

Figure 5.6: Perfect complements

$$x_1 = x_2 = x = \frac{m}{p_1 + p_2}.$$

The demand function for the optimal choice here is quite intuitive. Since the two goods are always consumed together, it is just as if the consumer were spending all of her money on a single good that had a price of $p_1 + p_2$.

Neutrals and Bads

In the case of a neutral good the consumer spends all of her money on the good she likes and doesn't purchase any of the neutral good. The same thing happens if one commodity is a bad. Thus, if commodity 1 is a good and commodity 2 is a bad, then the demand functions will be

$$x_1 = \frac{m}{p_1}$$
$$x_2 = 0.$$

Discrete Goods

Suppose that good 1 is a discrete good that is available only in integer units, while good 2 is money to be spent on everything else. If the consumer chooses 1, 2, 3, ... units of good 1, she will implicitly choose the consumption bundles $(1, m`p_1), (2, m`2p_1), (3, m`3p_1)$, and so on. We can simply compare the utility of each of these bundles to see which has the highest utility. We can use the indifference-curve analysis in Figure 5.7.

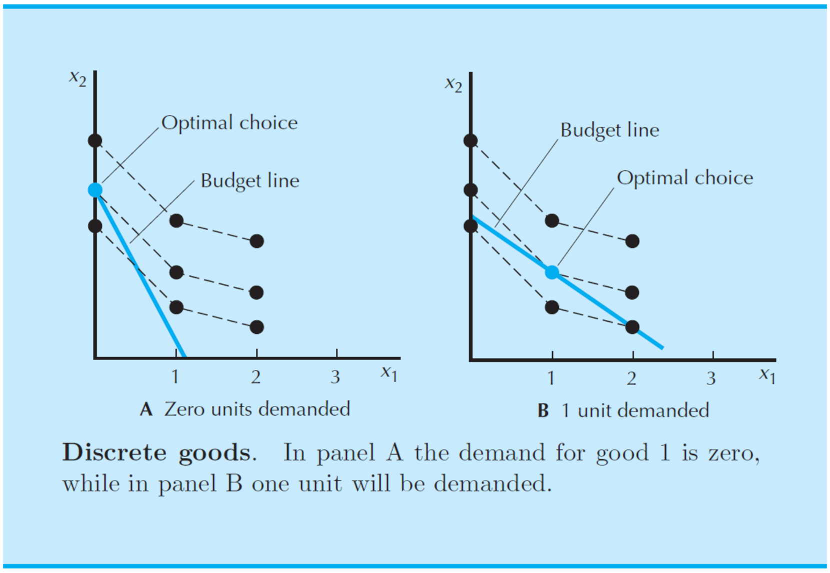


Figure 5.7: Discrete Goods

6 Technology

A technology is a process by which inputs are converted to an output. e.g. labor, a computer, a projector, electricity, and software are being combined to produce this lecture. Usually, several technologies will produce the same product a blackboard and chalk can be used instead of a computer and a projector.

6.1 Input Bundles and Outputs

x_i denotes the amount used of input i; i.e. the level of input i.
An input bundle is a vector of the input levels $(x_1, x_2, \cdots x_n)$.
y denotes the output level.

6.2 Production Functions

❖ Definition:6.1 Production Function ▽

The technology's production function states the maximum amount of output possible from an input bundle.

$$y = f(x_1, x_2, \cdots, x_n)$$

A production plan is an input bundle and an output level $(x_1, x_2, \cdots, x_n; y)$
A production plan is feasible if

$$y \le f(x_1, x_2, \cdots, x_n)$$

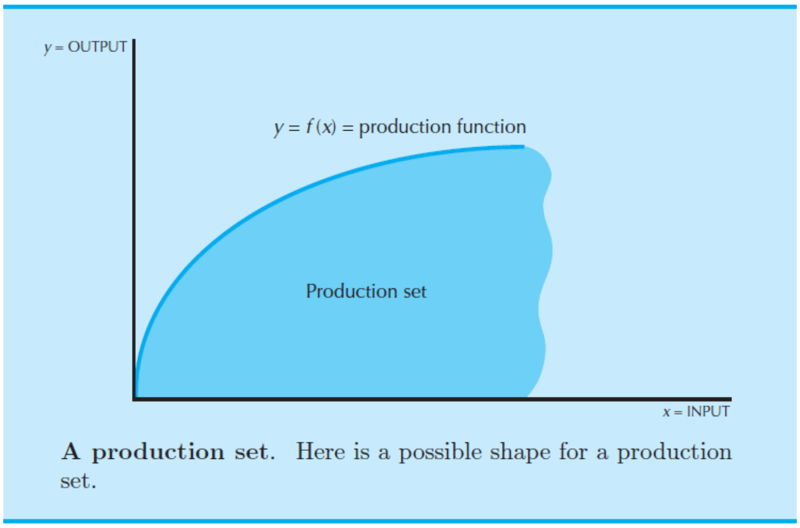


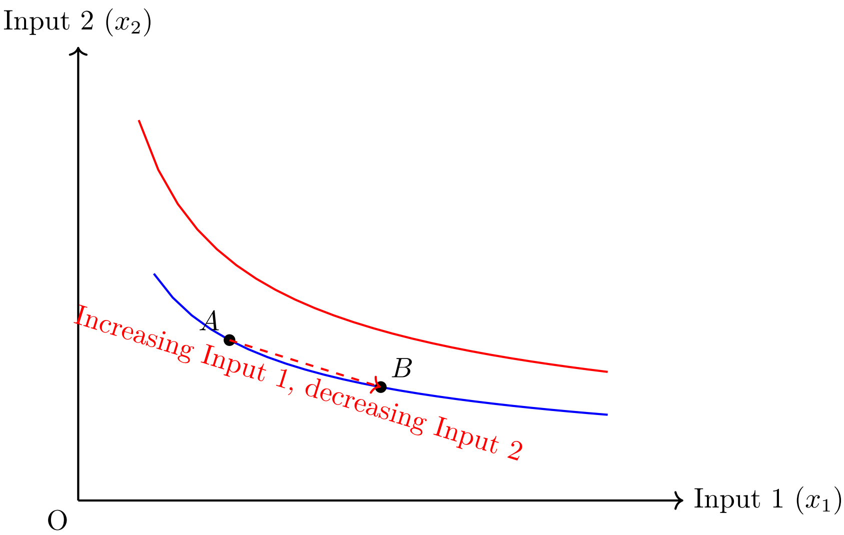
Figure 6.1: A Production Set

❖ Definition:6.2 Production Set ▽

The collection of all feasible production plans is the production set (生产集) or technology set (技术集).

❖ Definition:6.3 Isoquant ▽

An isoquant (等产量线) is the set of all possible combinations of inputs 1 and 2 that are just sufficient to produce a given amount of output.



6.3 Examples of Technology

Cobb-Douglas Technologies:

$$f(x_1, x_2) = A x_1^{\alpha} x_2^{\beta}.$$

Fixed-Proportions Technologies:

$$f(x_1, x_2) = \min(\alpha x_1, \beta x_2).$$

Perfect-Substitution Technologies:

$$f(x_1, x_2) = \alpha x_1 + \beta x_2.$$

CES Technologies:

$$f(x_1, x_2) = [\alpha x_1^{\rho} + (1-\alpha) x_2^{\rho}]^{\frac{1}{\rho}}.$$

6.4 Marginal Product

The marginal product (边际产品) of input i is the rate of change of the output level as the level of input i changes, holding all other input levels fixed. That is,

$$MP_1 = \frac{f(x_1 + \Delta x_1, x_2) - f(x_1, x_2)}{\Delta x_1} = \frac{\partial y}{\partial x_1}.$$

6.5 Returns to Scale

Marginal products describe the change in output level as a single input level changes.

Returns to scale describes how the output level changes as all input levels change in direct proportion (e.g. all input levels doubled, or halved).

If, for any input bundle

$$f(kx_1, kx_2, \cdots, kx_n) = kf(x_1, x_2, \cdots, x_n)$$

then the technology described by the production function f exhibits constant returns to scale (规模报酬不变,CRS).

6.6 Technical Rate of Substitution

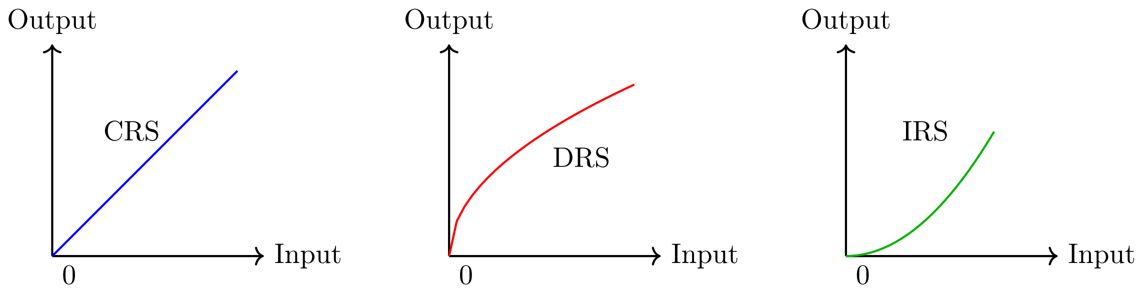
At what rate can a firm substitute one input for another without changing its output level?

The slope is the rate at which input 2 must be given up as input 1's level is increased so as not to change the output level. The slope of an isoquant is its technical rate of substitution.

Suppose $y = f(x_1, x_2) = x_1^a x_2^b$, so $\frac{\partial y}{\partial x_1} = a x_1^{a-1} x_2^b$ and $\frac{\partial y}{\partial x_2} = b x_1^a x_2^{b-1}$.

The technical rate of substitution is

$$\frac{\mathrm{d}x_2}{\mathrm{d}x_1} = -\frac{\partial y / \partial x_1}{\partial y / \partial x_2} = -\frac{a x_2}{b x_1}.$$



(a) Constant Returns to Scale (b) Decreasing Returns to Scale (c) Increasing Returns to Scale

Figure 6.2: Returns to Scale in Production

6.7 Elasticity of Substitution

❖ Definition:6.4 Elasticity of Substitution ▽

Elasticity of Substitution measures the ease with which one input can be substituted for another input in the production process, while keeping output constant.

$$\sigma = \frac{\mathrm{d}\ln(x_2/x_1)}{\mathrm{d}\ln|TRS|}.$$

If substitution elasticity is:

(1) high (like a linear production function, $\sigma \to \infty$), it means that the two inputs are easily interchangeable.

(2) moderate (like a Cobb Douglas production function, $\sigma = 1$), it means that the relative importance of inputs remains constant.

(3) low (like a Leontief production function, $\sigma = 0$)it means that substituting one input for another is difficult.

✎ Remark:6.5 ▽

A well behaved technology is **monotonic** and **convex**.

Monotonicity: More of any input generates more output.

Convexity: If the input bundles x' and x'' both provide y units of output then the mixture $ax' + (1a)x''$ provides at least y units of output, for any $0 < t < 1$.

♦ Theorem:6.6 ▽

If a production set is a **convex set**, then its production function is a **concave function**, vice versa.

Proof: A convex production set $Y = \{q, -x\}$, for any $y_1, y_2 \in Y$, we have

$$\alpha y_1 + (1-\alpha) y_2 \in Y, \quad \forall \alpha \in (0,1).$$

For production plan $y_i : (q = f(x_i), -x_i), \quad i = 1, 2$, define $y_i = f(x_i)$, we have

$$[\alpha f(x_1) + (1-\alpha) f(x_2), -(\alpha x_1 + (1-\alpha) x_2)] \in Y.$$

With restriction of production plan, we have $q \leq f(x)$, so that

$$\alpha f(x_1) + (1-\alpha) f(x_2) \leq f(\alpha x_1 + (1-\alpha) x_2).$$

□

7 Profit Maximization

7.1 Economic Profit

A firm uses inputs $j = 1, 2, \cdots, m$ to make products $i = 1, 2, \cdots, n$. Output levels are $y_1, y_2, \cdots, y_n$. Input levels are $x_1, x_2, \cdots, x_m$. Product prices are $p_1, p_2, \cdots, p_n$. Input prices are $w_1, w_2, \cdots, w_m$.

The economic profit generated by the production plan $(x_1, \cdots, x_m, y_1, \cdots, y_n)$ is

$$\Pi = p_1 y_1 + \cdots + p_n y_n - w_1 x_1 - \cdots - w_m x_m.$$

7.2 ISO-profit Curve

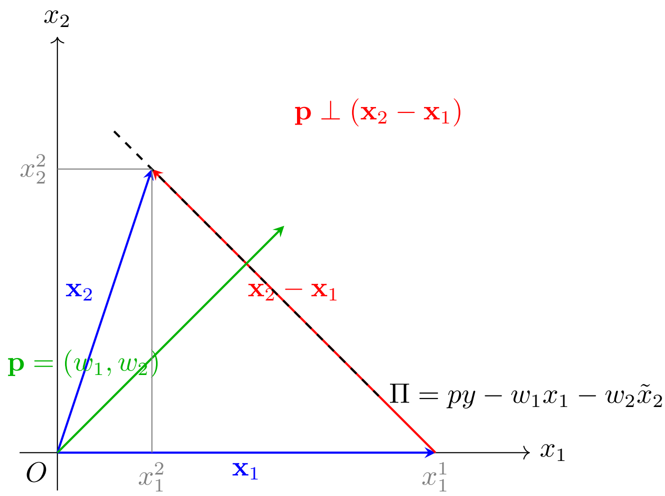
A Π iso-curve line contains all the production plans that yield a profit level of Π.

The equation of a Π iso-curve line is

$$\Pi = py - w_1 x_1 - w_2 \tilde{x}_2.$$

i.e.,

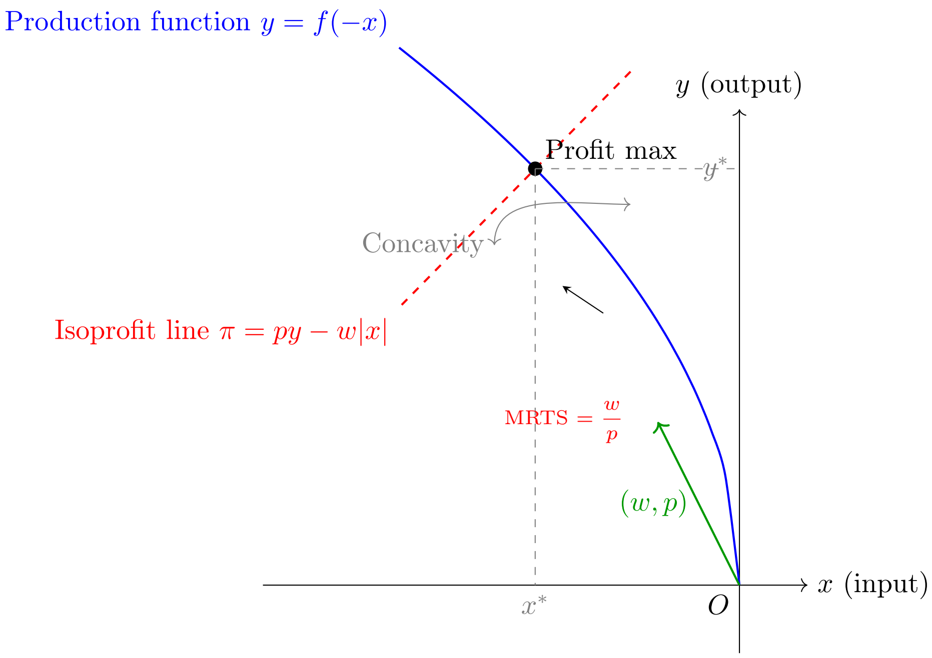
$$y = \frac{w_1}{p} x_1 + \frac{\Pi + w_2 \tilde{x}_2}{p}.$$



7.3 Profit Maximization

The firm's problem is to locate the production plan that attains the highest possible iso-profit line, given the firm's constraint on choices of production plans. The constraint is **production function**.

$$\max_x \quad pf(x) - wx$$
$$s.t. \quad f(x) = q$$



Suppose one input and one output. The profit-maximization problem is

$$\max_x \quad pf(x) - wx.$$

FOC is

$$0 = pf'(x) - w.$$

Rearrange to

$$\frac{w}{p} = MP = f'(x).$$

$MP \times p$ is the marginal revenue product of input 1, the rate at which revenue increases with the amount used of input 1.

If $MP \times p > w$, the profit increases with x.

If $MP \times p < w$, the profit decreases with x.

Now allow the firm to vary both input levels, i.e., both x_1 and x_2 are variable. The profit-maximization problem is

$$\max_{x_1, x_2} \quad pf(x_1, x_2) - w_1 x_1 - w_2 x_2.$$

FOCs are

$$p\frac{\partial f(x_1^*, x_2^*)}{\partial x_1} - w_1 = 0$$
$$p\frac{\partial f(x_1^*, x_2^*)}{\partial x_2} - w_2 = 0$$

Demand for inputs 1 and 2 can be solved as

$$x_1 = x_1(w_1, w_2, p)$$
$$x_2 = x_2(w_1, w_2, p)$$

7.4 Returns-to Scale and Profit Maximization

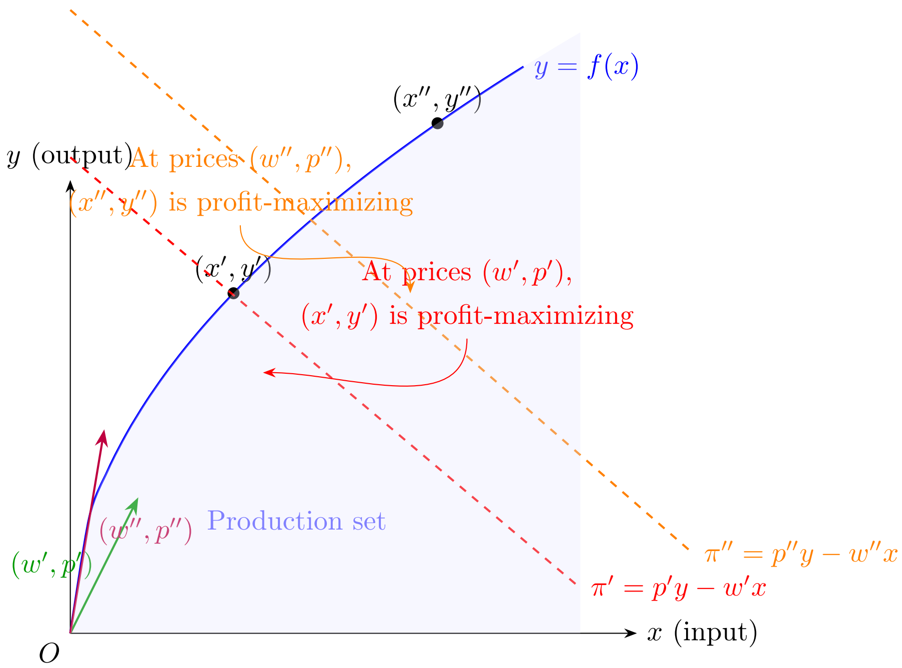
If a competitive firm's technology exhibits decreasing return-to-scale then the firm has a single long-run profit-maximizing production plan.

7.5 Revealed Profitability

Consider a competitive form with a technology that exhibits decreasing return-to scale.

For a variety of output and input prices we observe the firm's choices of production plans.

If a production plan (x', y') is chosen at price (w', p') we deduce that the plan (x', y') is revealed to be profit-maximizing for the prices (w', p').



7.6 Profit Maximization: An exercise

The production function is

$$y = x_1^{\alpha} x_2^{1-\alpha}.$$

Prices: p, w_1, w_2. Solve the profit-maximization problem.

$$\pi = p x_1^{\alpha} x_2^{1-\alpha} - w_1 x_1 - w_2 x_2$$

FOCs

$$\alpha p x_1^{\alpha-1} x_2^{1-\alpha} = w_1$$
$$(1-\alpha) p x_1^{\alpha} x_2^{-\alpha} = w_2$$

We have $\dfrac{w_1}{w_2} = \dfrac{\alpha}{1-\alpha}\dfrac{x_2}{x_1}$.
Rearrange, we have $x_2 = \dfrac{1-\alpha}{\alpha}\dfrac{w_1}{w_2}x_1$.
So that

$$\pi = \left[p\left(\frac{1-\alpha}{\alpha}\frac{w_1}{w_2}\right)^{1-\alpha} - \left(\frac{w_1}{\alpha}\right)\right] \cdot x_1 \triangleq k \cdot x_1.$$

♣ Corollary:7.1 ▽

If $k > 0$, then $x_1 \to \infty$.
If $k = 0$, then $x_1 \in (0, \infty)$.
If $k < 0$, then $x_1 = 0$.

Similarly, for x_2.

8 Cost Minimization

8.1 The Cost-Minimization Problem

Consider a firm using two inputs to make one output.

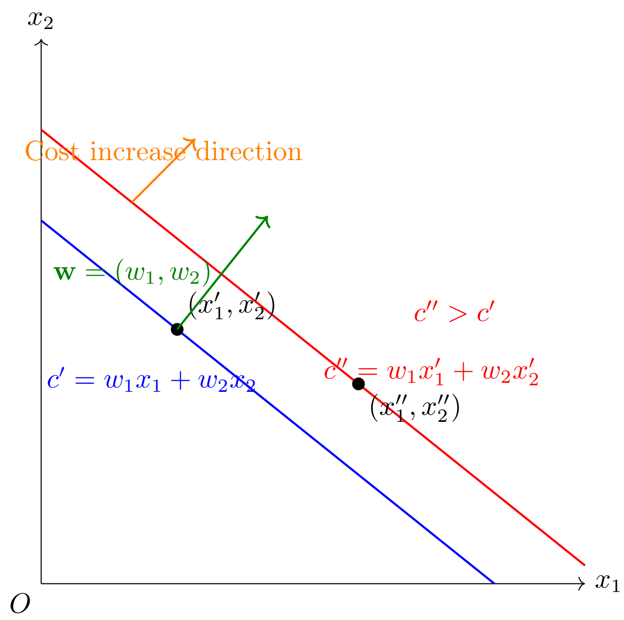
The production function is $y = f(x_1, x_2)$.

Take the output level $y \geq 0$ as given.

Given the input prices w_1 and w_2, the cost of an input bundle (x_1, x_2) is

$$w_1 x_1 + w_2 x_2.$$

8.2 ISO-Cost Curve



8.2.1 Cost-Minimization: An Exercise

The production function is

$$y = x_1^{1/3} x_2^{1/3}.$$

Prices: $p, w_1 = 1, w_2 = 2$. Solve the cost-minimization problem.

$$\min_{x_1, x_2} \quad x_1 + 2x_2$$

$$s.t. \quad f(x_1, x_2) = x_1^{1/3} x_2^{1/3} = q$$

$$\mathcal{L} = x_1 + 2x_2 - \lambda(x_1^{1/3} x_2^{1/3} - q).$$

FOCs

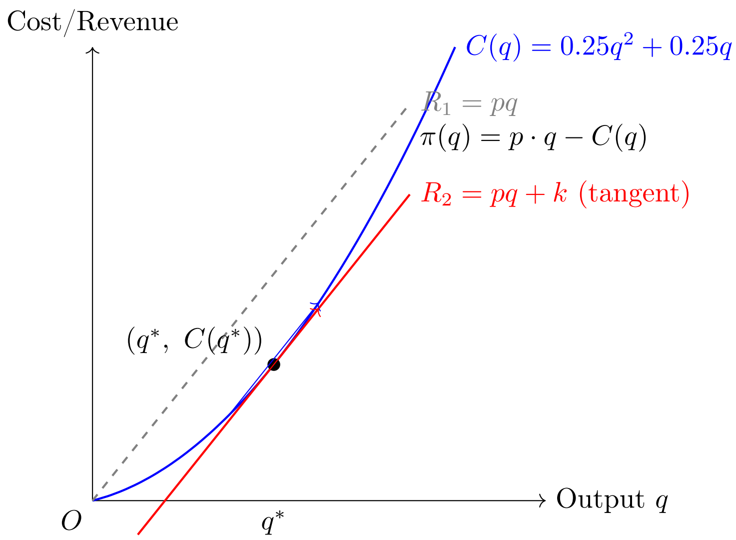
$$1 = \frac{1}{3}\lambda x_1^{-2/3} x_2^{1/3}$$
$$2 = \frac{1}{3}\lambda x_1^{1/3} x_2^{-2/3}$$
$$q = x_1^{1/3} x_2^{1/3}$$

We have $x_1 = \sqrt{2}q^{3/2}, \quad x_2 = \frac{1}{\sqrt{2}}q^{3/2}$.

So that $C(q) = 2\sqrt{2}q^{3/2}$.

$$\pi(q) = pq - 2\sqrt{2}q^{3/2} \Rightarrow q^* = \frac{p^2}{18}.$$

8.3 Returns-to Scale and Total Costs



8.4 Returns-to Scale and Average Costs

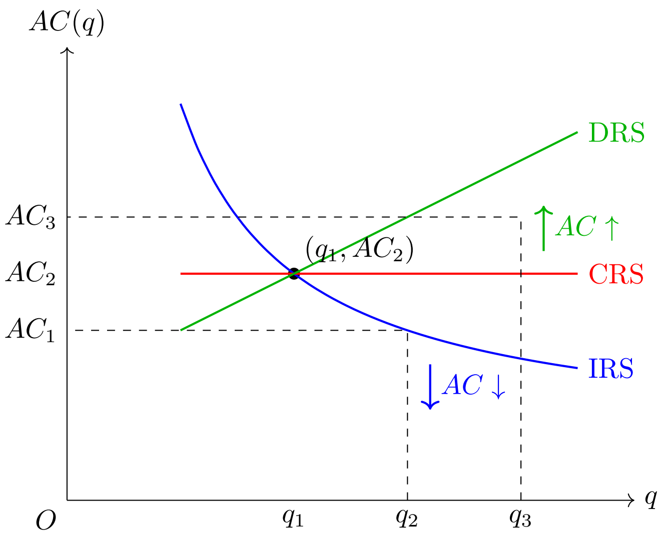
For positive output levels y, a firm's average total cost of producing y units is

$$AC = \frac{C(q)}{q}.$$

The returns-to-scale properties of a firm's technology determine how average production costs change with output level.

Our firm is presently producing y' output units.

How does the firm's average production cost change if it instead produces $2y'$ units of output?



8.4.1 Cost-Minimization: An Exercise (Perfect Substitute)

The production function

$$y = \alpha x_1 + \beta x_2.$$

Prices: p, w_1, w_2. Solve the cost-minimization problem.

$$\min_{x_1, x_2} \quad w_1 x_1 + w_2 x_2$$

$$s.t. \quad f(x_1, x_2) = \alpha x_1 + \beta x_2 \geq q$$

Rearrange, we have

$$w_1 x_1 + w_2 \frac{q - \alpha x_1}{\beta} \Rightarrow \left(w_1 - \frac{w_2}{\beta} \alpha \right) x_1 + \frac{q}{\beta} w_2.$$

Define $k = w_1 - \dfrac{w_2}{\beta}\alpha$,

if $k > 0$, then $x_1 = 0, x_2 = \frac{q}{\beta}$, $c_1 = q \cdot w_1/\alpha$,

if $k < 0$, then $x_1 = \frac{q}{\alpha}, x_2 = 0$, $c_2 = q \cdot w_2/\beta$,

if $k = 0$, whatever.

Then

$$C(q) = \min\{w_1/\alpha, w_2/\beta\} \cdot q.$$

8.4.2 Cost-Minimization: An Exercise (Perfect Complement)

The production function

$$y = \min(\alpha x_1, \beta x_2).$$

Prices: p, w_1, w_2. Solve the cost-minimization problem.

$$\min_{x_1, x_2} \quad w_1 x_1 + w_2 x_2$$

$$s.t. \quad f(x_1, x_2) = \alpha x_1 + \beta x_2 \geq q$$

9 Cost Curves

9.1 Fixed, Variable & Total Cost Functions

F is the total cost to a firm of its short-run fixed inputs. F, the firm's fixed cost, does not vary with the firm's output level.

VC is the total cost to a firm of its variable inputs when producing y output units. VC is the firm's variable cost function.

VC depends upon the levels of the fixed inputs.

TC is the total cost of all inputs, fixed and variable, when producing y output units. TC is the firm's total cost function.

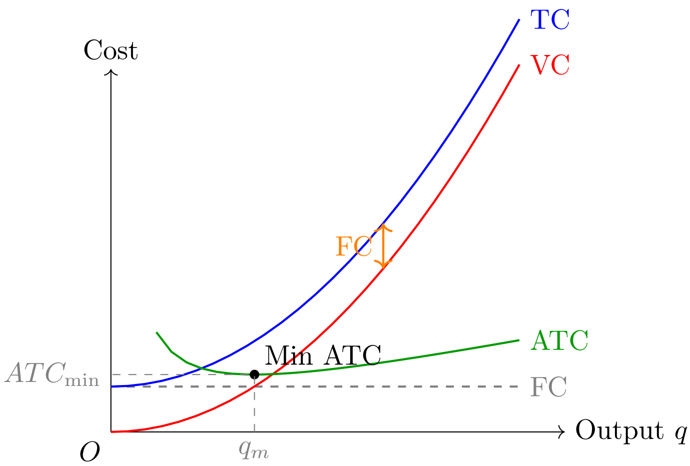
$$TC = F + VC, \quad c(y) = c_v(y) + F.$$

9.2 Types of Cost Curves

A total cost curve is the graph of a firm's total cost function.

A variable cost curve is the graph of a firm's variable cost function.

An average total cost curve is the graph of a firm's average total cost function.



9.3 Marginal Cost Function

Marginal cost is the rate-of-change of variable production cost as the output level changes. That is,

$$MC(y) = \frac{\partial c_v(y)}{\partial y}.$$

The firm's total cost function is

$$c(y) = c_v(y) + F$$

and the fixed cost F does not change with the output level y, so

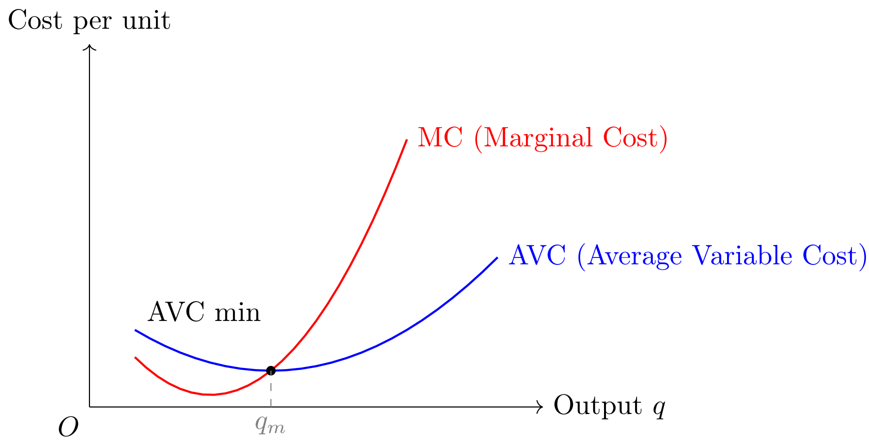
$$MC(y) = \frac{\partial c_v(y)}{\partial y} = \frac{\partial c(y)}{\partial y}.$$

MC is the slope of both the variable cost and the total cost functions.

9.4 Marginal Cost and Return to Scale

Q: When a firm operates under increasing returns to scale, is the marginal cost increasing or decreasing?

A: Marginal cost (MC) is decreasing under increasing returns to scale because production efficiency improves, reducing the cost per unit.



9.5 Short-Run & Long-Run Total Cost Curves

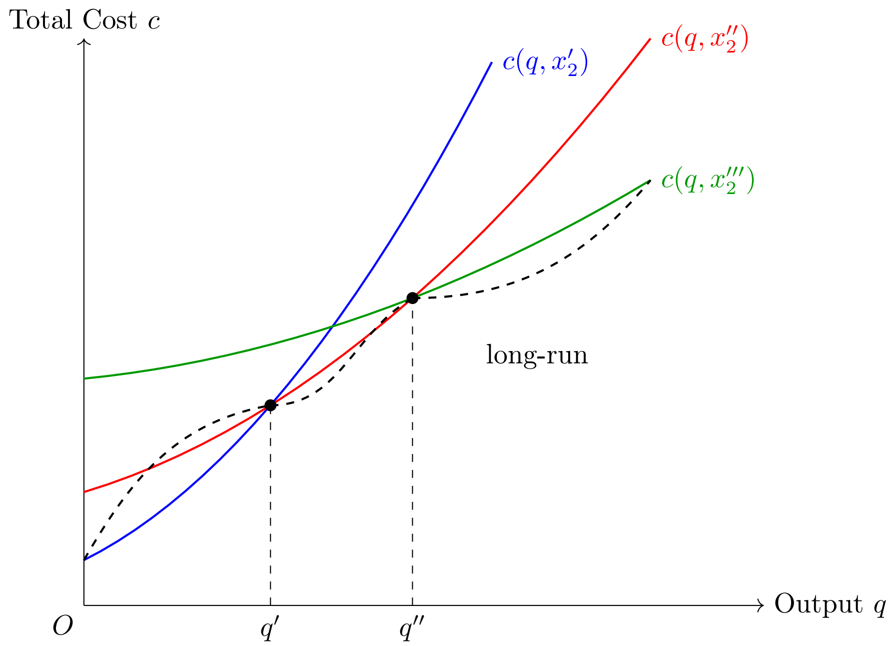
A firm has a different short run total cost curve for each possible short run circumstance. Suppose the firm can be in one of just three short runs ($x_2' < x_2'' < x_2'''$).

$$x_2 = x_2' \quad \text{or} \quad x_2 = x_2'' \quad \text{or} \quad x_2 = x_2'''.$$

The firm's long run total cost curve consists of the lowest parts of the short run total cost curves.

♣ Corollary:9.1 ▽

The long run total cost curve is the lower envelope of the short run total cost curves.



The production function is

$$y = x_1^{1/3} x_2^{2/3}.$$

Prices: p, w_1, w_2. Solve the short-run and long-run cost-minimization problem.
Short run:

$$\min_{x_1} \quad w_1 x_1 + w_2 \bar{x_2}$$
$$s.t. \quad f(x_1, x_2) = x_1^{1/3} x_2^{2/3} = q$$

Rearrange, we have $x_1 = q^3 x_2^{-2}$, so

$$C_S(q, \bar{x_2}) = w_1 q^3 \bar{x_2}^{-2} + w_2 \bar{x_2}.$$

Long run:

$$C_L(q) = \min_{x_2} \quad w_1 q^3 \bar{x_2}^{-2} + w_2 \bar{x_2}.$$

FOC:

$$C_L'(q) = -2w_1 q^3 x_2^{-3} + w_2 = 0 \Rightarrow x_2^* = \left(\frac{2w_1}{w_2}\right)^{\frac{1}{3}} q.$$

So that we have $C_L(q) = (2^{-2/3} + 2^{1/3}) w_1^{1/3} w_2^{2/3} q = (3 \times 2^{-2/3}) w_1^{1/3} w_2^{2/3} q.$

♣ Corollary:9.2 ▽

In general, for C-D production function, we have

$$C = \frac{w_1^{\alpha} w_2^{1-\alpha}}{(1-\alpha)^{1-\alpha} \alpha^{\alpha}} q.$$

A question: Is the marginal cost the same in the short and long run?

♣ **Corollary:9.3** ▽

Given q^*, MC is same in the short and long run.

Proof: Short run:

$$C_S(q^*, \bar{x_2}) = w_1(q^*)^3\bar{x_2}^{-2} + w_2\bar{x_2}.$$

With x_2^*,

$$\frac{\partial C_S(q, \bar{x_2})}{\partial q} = 3w_1(q^*)^2\bar{x_2}^{-2} = 3 \times 2^{-2/3}w_1^{1/3}w_2^{2/3}.$$

Long run:

$$C_L(q^*) = (3 \times 2^{-2/3})w_1^{1/3}w_2^{2/3}q^*.$$

$$MC_{q^*} = \left.\frac{\mathrm{d}C(q)}{\mathrm{d}q}\right|_{q=q^*} = 3 \times 2^{-2/3}w_1^{1/3}w_2^{2/3}.$$

□

10 Firm Supply

10.1 Market Environments

Pure competition (完全竞争): A firm in a perfectly competitive market knows it has no influence over the market price for its product. The firm is a market price taker.

Monopoly (垄断): Just one seller that determines the quantity supplied and the market clearing price.

Oligopoly (寡头垄断): A few firms, the decisions of each influencing the payoffs of the others.

Monopolistic Competition(垄断竞争): Many firms each making a slightly different product. Each firm's output level is small relative to the total.

10.2 The Firm's Short run Supply Decision (Perfect Competition)

Each firm is a profit maximizer and in a short run.

Each firm choose its output level by solving $\max_q \; pq - C(q)$.

10.3 Solve Firm Supply

10.3.1 Supply Curve

$$\begin{aligned} \max_{\mathbf{y}} \quad \mathbf{py} &\Rightarrow \max_{q,-x} \quad pq - wx \\ s.t. \quad \mathbf{y} \in Y &\Rightarrow s.t. \quad f(x) \geq q \end{aligned}$$

Step 1: Cost Minimization

$$\begin{aligned} \min_{x} \quad & wx \\ s.t. \quad & f(x) \geq q \end{aligned}$$

This step must has solution, we have cost function: $C(q)$.

Step 2: Profit Maximization

$$\max_{q} \quad pq - C(q)$$

It may have no solution.

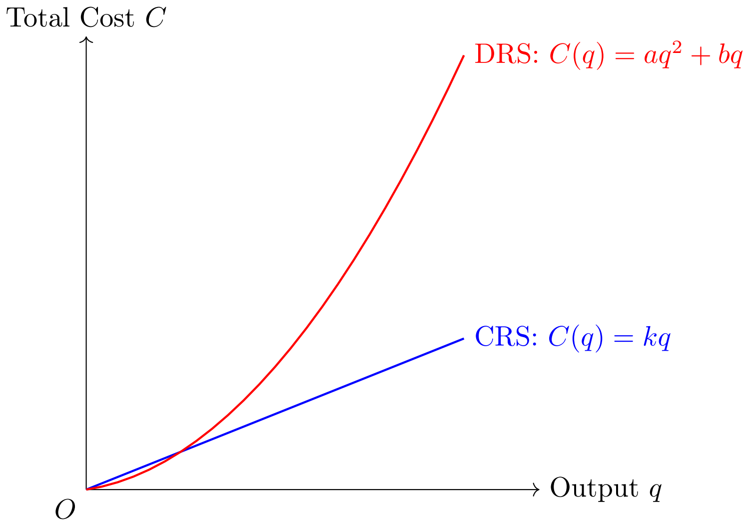
$$CRS \begin{cases} q \to \infty \\ q \in (0, \infty) \\ q = 0 \end{cases}$$

Step 3: FOC for Firm Decision

$$p = C'(q) \Rightarrow q(p) = C'^{-1}(p).$$

Without restriction, we take derivation directly.

$$q'(p) = \frac{1}{C''(p)}.$$



10.3.2 Short-Run Supply

Suppose $f(x_1, x_2) = \sqrt{x_1 x_2}$, with $\bar{x_2} = 5, w_1 = 1, w_2 = 1$.

$$FC = 5 \times w_2 = 5, \quad q = \sqrt{x_1 x_2} \Rightarrow x_1 = q^2/5.$$

$$C(q) = 5 + q^2/5, \quad \max \pi \Rightarrow \max\ pq - C(q).$$

FOC:

$$p = C'(q) = \frac{2}{5}q.$$

$$\pi = pq - C(q) = \frac{5}{2}p^2 - \left(5 + \frac{1}{5}\left(\frac{5}{2}p^2\right)\right) = \frac{5}{4}p^2 - 5.$$

✎ Remark:10.1 ▽

However, in the short-run, when price $p < AVC$, the firm choose to rest.

That is

$$p \geq AVC = \frac{q}{5} \geq 0.$$

So we have Supply Curve:

$$q = \frac{5}{2}p.$$

10.3.3 Long-Run Supply

Suppose $f(x_1, x_2) = \sqrt{x_1 x_2}$, with $x_2 = 5, w_1 = 1, w_2 = 1$.

$$C(q) = 5 + \frac{1}{5}q^2.$$

$$\pi = pq - C(q) = pq - (5 + \frac{1}{5}q^2), \quad \frac{\partial \pi}{\partial p} = 0 \Rightarrow q = \frac{5}{2}p.$$

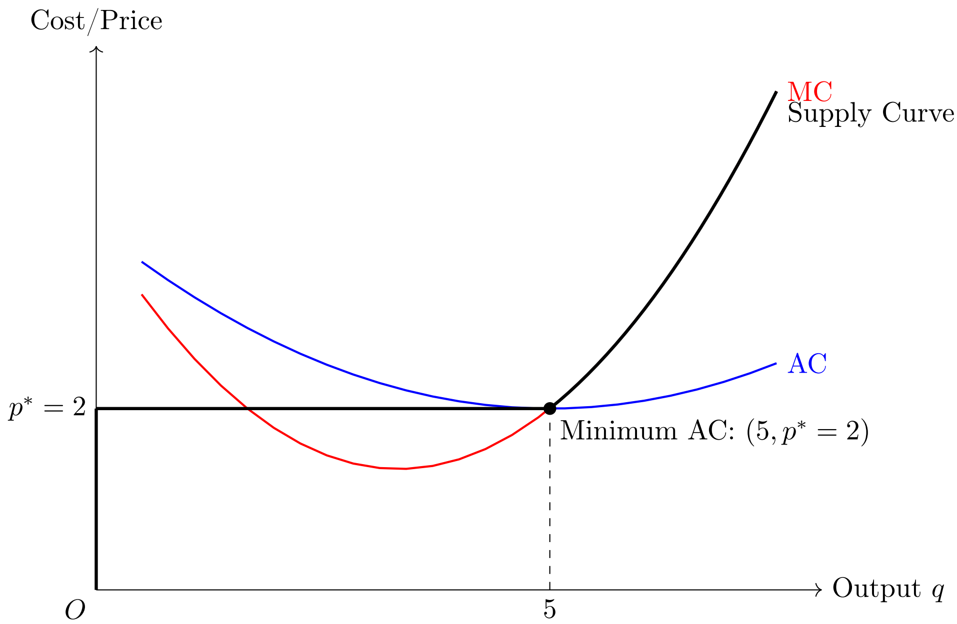
So that $\pi = \frac{5}{4}p^2 - 5$. When $p \geq 2$, the firm enter market, otherwise, not enter. The threshold $p = 2 \Rightarrow \min AC$.

$$AC = \frac{C(q)}{q} = \frac{5}{q} + \frac{q}{5}.$$

$$\text{FOC}: \frac{-5}{q^2} + \frac{1}{5} = 0 \Rightarrow q^* = 5.$$

Thus we have $AC_{\min} = 2$.

Supply curve:



11 Industry Supply

Firm supply q_i.

$$\max_{q_i} \quad pq_i - C(q_i)$$

Industry supply $Q(p) = \sum_{i=1}^{N} q_i(p), \quad i = 1, 2, \cdots, N$.

Suppose $f(x_1, x_2) = \sqrt{x_1 x_2}$, with $x_2 = 5, w_1^A = w_1^B = 1, w_2^A = 2 > w_2^B = 1$. We need to know $Q(p)$.

$$q^A(p) = \begin{cases} \frac{5}{2}p & \text{if } p \geq 2\sqrt{2} \\ 0 & \text{otherwise} \end{cases} \qquad q^B(p) = \begin{cases} \frac{5}{2}p & \text{if } p \geq 2 \\ 0 & \text{otherwise} \end{cases}$$

Thus, we have $Q(p) = \begin{cases} 5p & \text{if } p \geq 2\sqrt{2} \\ \frac{5}{2}p & \text{if } 2 \leq p < 2\sqrt{2} \\ 0 & \text{otherwise} \end{cases}$

11.1 Short-Run Industry Supply

In the short-run, the number of firms in the industry is, temporarily, fixed. We can put market demand curve with industry supply curve together to get equilibrium price and quantity.

Suppose $Q^d(p) = 10 - 2p$, we have $q^A = 0,\ q^B = 50/9,\ p = 20/9$.

11.2 Long-Run Industry Supply

In the long-run every firm now in the industry is free to exit and firms now outside the industry are free to enter.

The industry's long run supply function must account for entry and exit as well as for the supply choices of firms that choose to be in the industry.

> ✎ **Remark:11.1** ▽
>
> Positive economic profit induces entry.

Economic profit is positive when the market price p_s^e is higher than a firm's minimum average total cost

$$p_S^e > \min AC(y).$$

Entry increases industry supply, causing p_s^e to fall.

Suppose $f(x_1, x_2) = \sqrt{x_1, x_2}$, with $x_2 = 5, w_1 = w_2 = 1$.

We've known that now the equilibrium price $p^* = 20/9 > 2$, so firm will enter the market for a positive profit, with more and more firms enter the market, q rise and p fall, until $p^{**} = 2$. So that $p^{**} = 2 = \min AC(q)$ with every firm produce $q = 5$.

If there is a demand restriction $Q^d = 10 - 2p = 6$, we have

$$N = \frac{Q^d}{q} = 1.2 \approx 1.$$

If there is not a demand restriction, we are not sure that N is infty or not. The industrial supply is almost a horizontal line, with $S_i = S_1/2^{i-1}$.

12 Monopoly

Suppose that the monopolist seeks to maximize its economic profit,

$$\max p(q)q - C(q).$$

Marginal revenue is the rate of change of revenue $p(q)q$ as the output level q increases

$$MR(q) = p'(q)q + p(q).$$

Marginal cost is the rate of change of total cost as the output level y increases

$$MC(q) = C'(q).$$

FOC($MR = MC$):

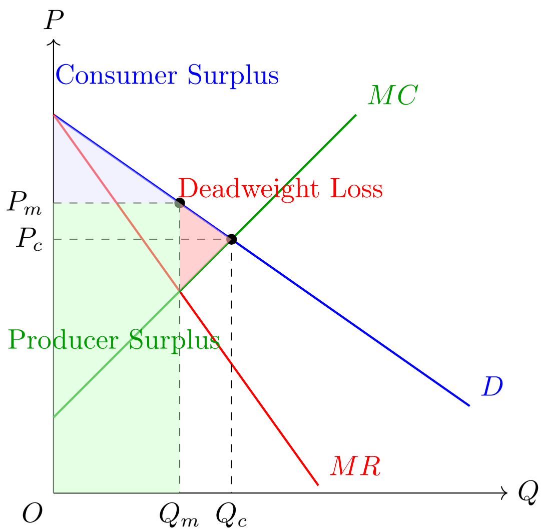
$$\begin{aligned} p'(q)q + p(q) &= C'(q) \\ \Rightarrow p(q)\left[1 + \frac{p'(q)q}{p(q)}\right] &= C'(q) \\ p(q)\left[1 + \frac{1}{\varepsilon}\right] &= C'(q) \end{aligned}$$

From $1 + \frac{1}{\varepsilon} < 1$, we have $p(q) > MC$.
In a competitive market $\varepsilon \to -\infty$, so $p(q) = MC$.
We have $Q_{\text{monopoly}} < Q_{\text{competition}}$, when $|\varepsilon|$ rise, $Q_{\text{monopoly}} \to Q_{\text{competition}}$.

✎ **Remark:12.1** ▽

Normally, we have $1 + \frac{1}{\varepsilon} \geq 0$, because when $\varepsilon < -1$, the goods are more elastic, when prices fell slightly, and demand rose sharply, and firms make more profit. A rational firm will change p and q to make $\varepsilon < -1$.

Monopoly is inefficient, because it will cause deadweight loss.



An example: To maximize the profits, Suppose

$$p(q) = a - bq$$
$$c(q) = F + \alpha q + \beta q^2$$

From $MR = MC$, we have $a - 2bq = \alpha + 2\beta q$, so that

$$\begin{cases} q^* &= \dfrac{a-\alpha}{2(b+\beta)} \\ p(q^*) &= a - b \cdot \dfrac{a-\alpha}{2(b+\beta)} \end{cases}$$

$$p(q) = \frac{\varepsilon}{\varepsilon+1} MC \Rightarrow \frac{\mathrm{d}p}{\mathrm{d}MC} = \frac{\varepsilon}{\varepsilon+1} > 1.$$

12.1 A Profits Tax Levied on a Monopoly

A profits tax levied at rate t reduces profit from $\pi(q^*)$ to $(1-t)\pi(q^*)$.

Q: How is after tax profit $(1-t)\pi(q^*)$?

A: By maximizing before tax profit, $\pi(q^*)$.

So a profits tax has no effect on the monopolist's choices of output level, output price, or demands for inputs.

♣ **Corollary:12.2** ▽

I.e., the profits tax is a neutral tax.

12.2 Quantity Tax Levied on a Monopolist

A quantity tax of t/output unit raises the marginal cost of production by t. So the tax reduces the profit maximizing output level, causes the market price to rise, and input demands to fall.

♣ **Corollary:12.3** ▽

The quantity tax is distortionary.

12.3 Natural Monopoly

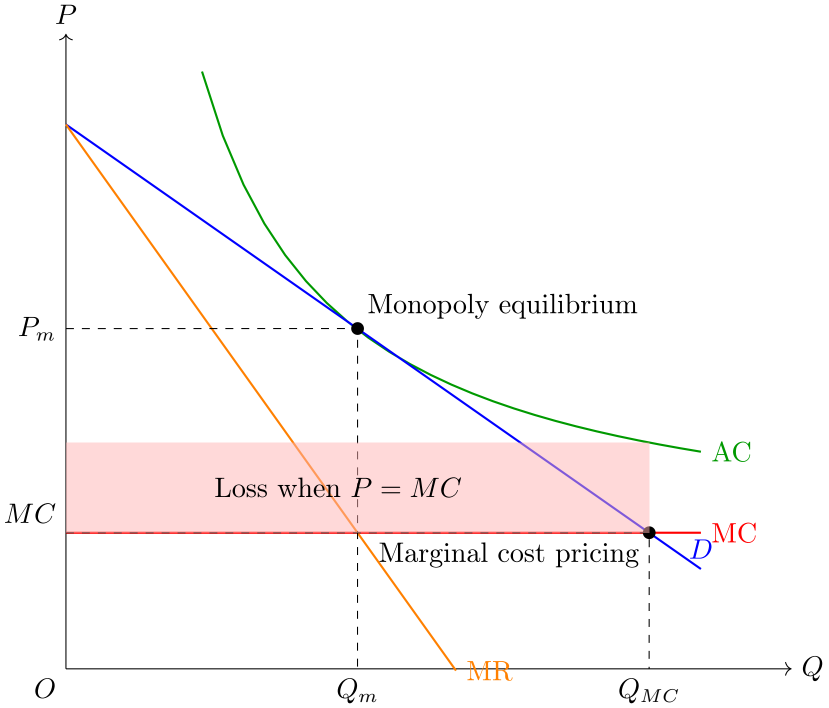
❖ **Definition:12.4 Natural Monopoly** ▽

A natural monopoly arises when the firm's technology has economies of scale large enough for it to supply the whole market at a lower average total production cost than is possible with more than one firm in the market.

Cases: Water, electricity, and gas industries.

At the efficient output level Q_{MC}, $AC(Q_{MC}) > MC(Q_{MC})$ so the firm makes an economic loss.

So a natural monopoly cannot be forced to use marginal cost pricing. Doing so makes the firm exit, destroying both the market and any gains to trade. Regulatory schemes can induce the natural monopolist to produce the efficient output level without exiting.



13 Monopoly Behavior

So far a monopoly has been thought of as a firm which has to sell its product at the same price to every customer. This is uniform pricing.

Price discrimination can earn a monopoly higher profits.

13.1 First Degree Price Discrimination

❖ Definition:13.1 First Degree Price Discrimination ▽

Each output unit is sold at a different price. Price may differ across buyers.

It requires that the monopolist can discover the buyer with the highest valuation of its product, the buyer with the next highest valuation, and so on.

For a monopolist, the maximization problem is

$$\begin{aligned} &\max_{r,q} \quad r - C(q) \\ &s.t. \quad v(q) \geq r \end{aligned}$$

where r is the consumer's payment, $v(q)$ is the value of q goods for consumer.

FOCs: $v'(q^*) = c'(q^*) = p, \quad r = v(q^*)$.

When $r = PS + CS = PS$, the first degree price discrimination realized, $\pi = r - C(q^*)$.

13.2 Second Degree Price Discrimination

❖ Definition:13.2 Second Degree Price Discrimination ▽

Non-linear pricing: Unit price depends on quantity purchased, Bulk discount.

Setting: A seller does not know the willingness to pay by each individual buyer. Consumer's marginal willingness to pay declines with quantity

Setting a uniform price is not optimal: Too high a price would lose high volume consumer. Too low a price would lost revenue from low volume consumer.

Mechanism: Set price for different volumes to let consumers identify themselves.

13.3 Third Degree Price Discrimination

❖ Definition:13.3 Third Degree Price Discrimination ▽

Price paid by buyers in a given group is the same for all units purchased. But price may differ across buyer groups.

Quality of goods is the same across groups.

Can identify groups but no further identification within that group.

A monopolist manipulates market price by altering the quantity of product supplied to that market.

Two markets, 1 and 2. y_1 is the quantity supplied to market 1. Market 1's inverse demand function is $p_1(q_1)$.

y_2 is the quantity supplied to market 2. Market 2's inverse demand function is $p_2(q_2)$.

For given supply levels y_1 and y_2 the firm's profit is

$$\pi(q_1, q_2) = p_1(q_1)q_1 + p_2(q_2)q_2 - c(q_1 + q_2)$$

FOCs:

$$\frac{\partial \pi}{\partial q_1} = \frac{\partial (p_1(q_1)q_1)}{\partial q_1} - \frac{\partial C(q_1 + q_2)}{\partial (q_1 + q_2)} \times \frac{\partial (q_1 + q_2)}{\partial q_1} = 0$$
$$\frac{\partial \pi}{\partial q_2} = \frac{\partial (p_2(q_2)q_2)}{\partial q_2} - \frac{\partial C(q_1 + q_2)}{\partial (q_1 + q_2)} \times \frac{\partial (q_1 + q_2)}{\partial q_2} = 0$$

Thus we have

$$MR_1(q_1) = MR_2(q_2) = \frac{\partial (p_1(q_1)q_1)}{\partial q_1} = \frac{\partial (p_2(q_2)q_2)}{\partial q_2} = \frac{\partial C(q_1 + q_2)}{\partial (q_1 + q_2)}.$$

And

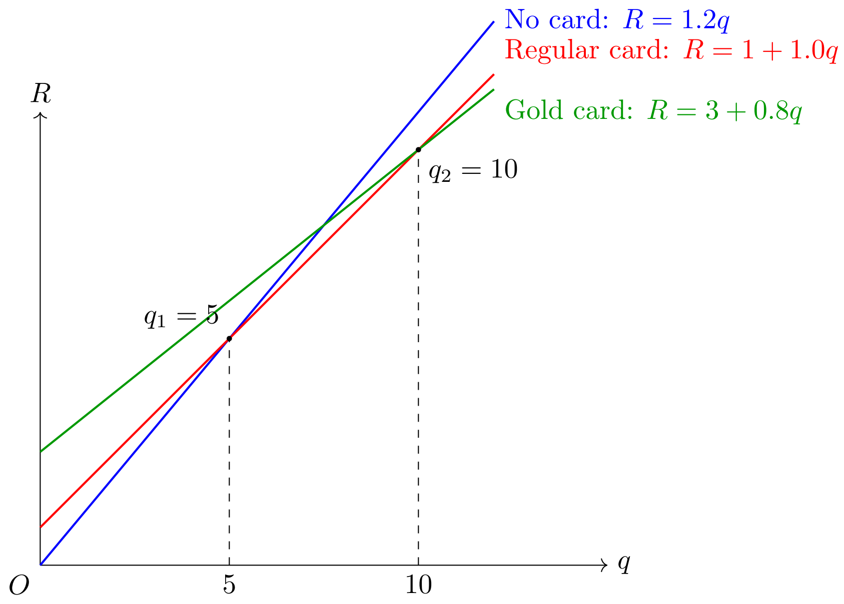
$$p_1(q_1)\left[1 + \frac{1}{\varepsilon_1}\right] = p_2(q_2)\left[1 + \frac{1}{\varepsilon_2}\right].$$

Therefore $p_1(q_1^*) > p_2(q_2^*)$ only if $1 + \frac{1}{\varepsilon_1} < 1 + \frac{1}{\varepsilon_2} \Rightarrow \varepsilon_1 > \varepsilon_2$, which means that good 1 is less elastic and good 2 is more elastic.

✎ **Remark:13.4** ▽

Summary: Price setting is related to the elasticity of demand. When the price elasticity of demand is high, consumers are more sensitive to prices, and by setting low prices, they can attract more consumers and increase profits.

13.4 Two-Part Tariffs



13.5 An Exercise

Suppose two types of consumers in the market. 10 of them $Q_1 = 100 - 2p$, another 10 of them $Q_2 = 50 - 2p$.

The firm's marginal cost is 1. i.e., $C(q) = q$.

Case 1: cannot price discrimination (uniform price)

We have

$$\begin{cases} Q_1, Q_2 = 0 & p > 50 \\ Q_1 > 0, Q_2 = 0 & 25 \le p \le 50 \\ Q_1, Q_2 > 0 & 1 < p < 25 \end{cases}$$

If $p \in [25, 50]$:

$$\max_p \quad (p-1)[100-2p] \times 10$$
$$s.t. \quad p \in [25, 50]$$

We have $p^* = 25.5$ and $\pi^1 = 24.5 \times 10 \times 49 = 12005$.

If $p \in (1, 25)$:

$$\max_p \quad (p-1)\{[100-2p] \times 10 + [50-2p] \times 10\}$$
$$s.t. \quad p \in (1, 25)$$

We have $p^* = 19.25$ and $\pi^2 = 13322.5$.

So that $\pi^2 > \pi^1$.

Case 2: Third-degree price discrimination

Need to satisfy $MR_1 = MR_2 = MC = 1$.
From $R_1 = P_1 Q_1 = 50Q_1 - \frac{1}{2}Q_1^2 \Rightarrow MR_1 = 50 - Q_1$.
Similarly, $MR_2 = 25 - Q_2$.
We have

$$\begin{cases} Q_1 = 49 \\ Q_2 = 24 \end{cases} \Rightarrow \begin{cases} p_1 = 25.5 \\ p_2 = 13 \end{cases}$$

Compare to $p = 77/4$ in Case 1, p_2 is lower, implying a higher demand-price elasticity in this market.

We have $\pi^3 = 14885$.

Case 3: Two-part tariffs

$$\max_{T,p} \quad 20 \times T + (p-1)[(100-2p) \times 10 + (50-2p) \times 10]$$
$$s.t. \quad CS_1(p) = (50-p)^2 \geq T$$
$$CS_2(p) = (25-p)^2 \geq T$$

If $T = (25-p)^2, CS_1 \geq T$, $\max_p 20 \times (25-p)^2 + (p-1)(1500-40p)$.
We have $p = 13.5 < 77/4$, $T = 132.25$, $\pi^4 = 14645$.
If $T = (50-p)^2$, $\max_p 10 \times (50-p)^2 + (p-1)(1000-20p)$.
We have $p = 1$, $T = 2401$, $\pi^5 = 24000$.

✎ **Remark:13.5** ▽

Note: T can vary with price p, usually a lower p corresponds to a higher T.

14 Oligopoly

A monopoly is an industry consisting a single firm.

A duopoly is an industry consisting of two firms.

❖ Definition:14.1 Oligopoly ▽

An oligopoly is an industry consisting of a few firms. Particularly, each firm's own price or output decisions affect its competitors' profits.

We analyze markets in which the supplying industry is oligopolistic by considering the duopolistic case of two firms supplying the same product.

14.1 Quantity Competition

Assume that firms compete by choosing output levels.

If firm 1 produces q_1 units and firm 2 produces q_2 units then total quantity supplied is $q_1 + q_2$. The market price will be $p(q_1 + q_2)$.

The firms' total cost functions are $C_1(p_1)$ and $C_2(p_2)$.

Suppose firm 1 takes firm 2's output level choice q_2 as given. Then firm 1 sees its profit function as

$$\pi_1(q_1; q_2) = p(q_1 + q_2)q_1 - C_1(q_1)$$

Given q_2, what output level y_1 maximizes firm 1's profit?

Suppose that the market inverse demand function is

$$p(Q) = 60 - Q.$$

and that the firms' total cost functions are

$$C_1(q_1) = q_1^2 \quad \text{and} \quad C_2(q_2) = 15q_2 + q_2^2.$$

Then, for given q_2, firm 1's profit function is

$$\pi_1(q_1; q_2) = (60 - q_1 - q_2)q_1 - q_1^2.$$

So, given q_2, firm 1's profit maximizing output level solves (i.e., firm 1's best response to q_2):

$$q_1 = R_1(q_2) = 15 - q_2/4.$$

Similarly, we have

$$q_2 = R_2(q_1) = 45/4 - q_1/4.$$

Solve the reaction curves.

Firm 1:

$$\pi(q_1, q_2) = (60 - q_1 - q_2)q_1 - q_1^2.$$

Profit max, we have

$$q_1^* = 15 - \frac{1}{4}q_2.$$

Firm 2: Similarly, we have

$$q_2^* = \frac{45}{4} - \frac{q_1}{4}.$$

14.2 Cournot-Nash Equilibrium

With

$$q_1^* = R_1(q_2^*) \quad \text{and} \quad q_2^* = R_2(q_1^*)$$

We have equilibrium $(q_1^*, q_2^*) = (13, 8)$.

Thus

$$\begin{cases} p = 60 - Q = 39 \\ \pi_1^C = 338 \\ \pi_2^C = 128 \end{cases}.$$

✎ **Remark:14.2** ▽

Generally, given firm 2's chosen output level q_2, firm 1's profit function is

$$\pi_1(q_1; q_2) = p(q_1 + q_2)q_1 - C_1(q_1)$$

and the profit-maximizing value of q_1 solves

$$\frac{\partial \pi_1}{\partial q_1} = p(q_1 + q_2) + q_1 \frac{\partial p(q_1 + q_2)}{\partial q_1} - C_1'(q_1) = 0.$$

The solution, $q_1 = R_1(q_2)$, is **firm 1's Cournot-Nash reaction to** q_2.
Similarly, $q_2 = R_2(q_1)$, is **firm 2's Cournot-Nash reaction to** q_1.

14.3 Collusion

The Cournot-Nash equilibrium not profits the largest that the firms can earn in total. There are profit incentives for both firms to "cooperate" by lowering their output levels. This is **collusion**.

Firms that collude are said to have formed a **cartel**. If firms form a cartel:

$$\begin{aligned} \max_{q_1, q_2} \quad & \pi_1 + \pi_2 \\ s.t. \quad & \pi_1 \geq \pi_1^C = 338 \\ & \pi_2 \geq \pi_2^C = 128 \end{aligned}$$

FOCs:

$$MC_1 = MC_2 \qquad Q = Q_{\text{monopoly}}$$

We have

$$\begin{cases} q_1 = 12.5 \Rightarrow \pi_1 = 375 \\ q_2 = 5 \Rightarrow \pi_2 = 112.5 < \pi_2^C \\ p = 42.5 \end{cases}$$

Not a collusion equilibrium.

14.4 Some More General Cases

Suppose N firms in the market , they have the same $MC = C$. The inverse demand function $P = A - b(Q)$.

(A) Market power

$$Q = \sum_{i=1}^{N} q_i \Rightarrow \pi = p(Q)q_j - c_j(Q_j)$$

$$\max_{q_j} \pi \Rightarrow p(Q) + p'(Q)q_j - c_j'(q_j) = 0$$

Thus we have

$$p(Q)\left[1 + \frac{p'(Q)Q}{p(Q)}\frac{q_j}{Q}\right] = c_j'(q_j).$$

Where $\frac{p'(Q)Q}{p(Q)}$ is 1/elasticity of demand, write as price ε, $\frac{q_j}{Q}$ is the market share S_j.
So

$$p(q)\left[1 + \frac{S_j}{\varepsilon}\right] = c_j'(q_j).$$

$$\begin{cases} S_j = 1 & , \quad \text{monopoly, large market power} \\ S_j = 0 & , \quad \text{perfect competition, no market power} \\ S_j \in (0,1) & , \quad \text{cournot competition} \end{cases}$$

(B) Find each firm's q_j

$$\pi_j = \left[A - b\sum_{i-1}^{N} q_i - C\right] q_j, \quad \max \pi_j.$$

FOC:

$$\left(A - b\sum_{i-1}^{N} q_i - C\right) - bq_j = 0.$$

From $MC = C \Rightarrow q_j = q$, we have $A - b(N+1)q - C = 0$.

So that

$$\begin{cases} q = \dfrac{A-C}{(N+1)b} \\ \\ \pi = \dfrac{(A-C)^2}{(N+1)^2 b} \end{cases}.$$

(C) With fixed cost

Suppose $C_j(q_j) = c_j(q_j) + F$, firm entry condition $\pi > 0$.

$$\begin{aligned} N = 1: \quad \pi(1) &= \frac{(A-C)^2}{4b} - F \\ N = 2: \quad \pi(2) &= \frac{(A-C)^2}{9b} - F \\ &\cdots \\ N = N: \quad \pi(N) &= \frac{(A-C)^2}{(N+1)^2 b} - F = 0 \\ \Rightarrow N = \frac{A-C}{\sqrt{bF}} - 1. \end{aligned}$$

15 General Equilibrium and Exchange

15.1 General Equilibrium

How are prices and quantities simultaneously determined in an economy.(i.e., all markets)

We simplify analysis (2×2 model):

2 goods (x, y), 2 consumers (A, B), and endowments (w^x, w^y).

Consumers' utility: $u_A(x_A, y_A), u_B(x_B, y_B)$.

Initial endowments of x and y. For A: (w_A^x, w_A^y);For B: (w_B^x, w_B^y).

An allocation $\{(x_A, y_A), (x_B, y_B)\}$ is feasible if

For good x: $x_A + x_B = w_A^x + w_B^x = w^x$.

For good y: $y_A + y_B = w_A^y + w_B^y = w^y$.

✎ Remark:15.1 ▽

Intuition: Demand=Supply in both markets (Market clearing conditions).

In general, equilibrium will be efficient, but, how to formalize?

❖ Definition:15.2 Pareto Improvement ▽

It makes at least one person better off, and nobody worse off.

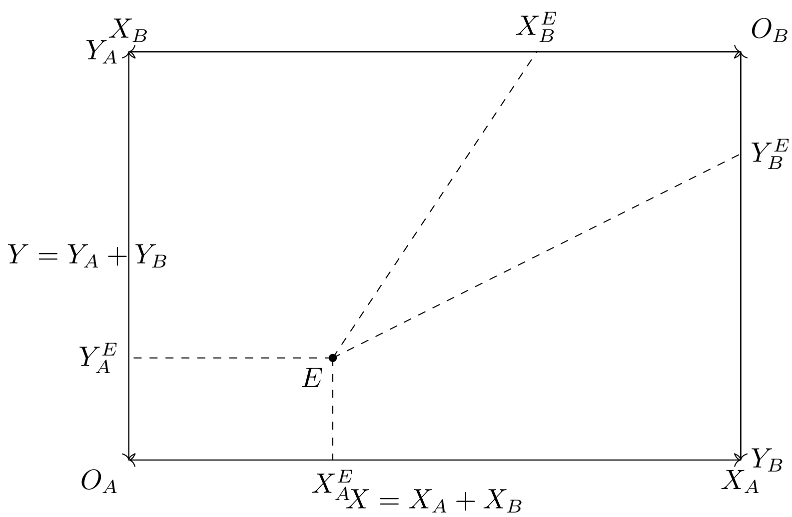
❖ Definition:15.3 Pareto Efficiency ▽

(1) Feasible;

(2) No further pareto improvement possible.

For any interior Pareto Efficient allocation: $MRS_A = MRS_B$.

Contract curve: Set of all Pareto Efficient allocation.



15.2 Contract Curve

Suppose $u_A = u_B = \sqrt{x_1 x_2}$, $w_A = (6,4)$, $w_B = (2,2)$.
Derive a single equation for the contract curve.

$$\begin{aligned} \max \quad & u_A = \sqrt{x_1^A x_2^A} \\ s.t. \quad & u_B = \sqrt{(8 - x_1^A)(6 - x_2^A)} = \tilde{u} \in [0, 4\sqrt{3}] \end{aligned}$$

$$\mathcal{L} = \sqrt{x_1^A x_2^A} - \lambda \left[\tilde{u} - \sqrt{(8 - x_1^A)(6 - x_2^A)} \right].$$

FOCs:

$$\begin{cases} \frac{1}{2}\sqrt{\frac{x_2^A}{x_1^A}} = -\lambda \sqrt{\frac{6 - x_2^A}{8 - x_1^A}} \\ \frac{1}{2}\sqrt{\frac{x_1^A}{x_2^A}} = -\lambda \sqrt{\frac{8 - x_1^A}{6 - x_2^A}} \end{cases} \Rightarrow \frac{x_2^A}{x_1^A} = \frac{3}{4}$$

15.3 Competitive equilibrium

Set of prices (p_x^*, p_y^*) and allocation $(x_A, y_A), (x_B, y_B)$ such that:
(1)All consumers maximize utility.
(2)Markets clear (Demand = Supply) in aggregate terms.

♦ **Theorem:15.4 First Welfare Theorem** ▽
Every competitive equilibrium (CE) is Pareto Efficient (PE).

In other words: Allowing free market competition can maximize efficiency.

♦ **Theorem:15.5 Second Welfare Theorem** ▽
Can set prices and endowments such that any Pareto Efficient allocation (PE) is a competitive equilibrium (CE).

Suppose $p_1, p_2 = 1$ (normalized), $u_A = u_B = \sqrt{x_1 x_2}$, $w^A = (6,4)$, $w^B = (2,2)$.
Max A, B utility.
For A,

$$\begin{aligned} \max \quad & \sqrt{x_1^A x_2^A} \\ s.t. \quad & p_1 x_1^A + x_2^A = 6p_1 + 4 \end{aligned}$$

For B,

$$\begin{aligned} \max \quad & \sqrt{x_1^B x_2^B} \\ s.t. \quad & p_1 x_1^B + x_2^B = 2p_1 + 2 \end{aligned}$$

Market clear, for A,

$$\begin{cases} x_1^A + x_1^B = 8 \\ x_2^A + x_2^B = 6 \end{cases} \quad \text{(colinear)} \ \Rightarrow \begin{cases} \frac{1}{2}\sqrt{\frac{x_2^A}{x_1^A}} = \lambda p_1 \\ \frac{1}{2}\sqrt{\frac{x_1^A}{x_2^A}} = \lambda \end{cases} \quad \Rightarrow \frac{x_2^A}{x_1^A} = p_1$$

Substitute x_2 in the *s.t.*, we have

$$\begin{cases} 2p_1 x_1^A = 6p_1 - 4 \\ 2p_1 x_1^B = 2p_1 + 2 \end{cases} \quad \Rightarrow p_1(x_1^A + x_1^B) = 4p_1 + 3$$

Use market clear condition, $p_1 = \frac{3}{4}$, and with *s.t.*, we have

$$\begin{cases} x_1^A = \frac{17}{3} \qquad x_2^A = \frac{17}{4} \\ \\ x_1^B = \frac{7}{3} \qquad x_2^B = \frac{7}{4} \end{cases}$$